Problem Set 2: Squares and Integers

Opens: 3 p.m. October 25th, 2021

Due: 3 p.m. November 8th, 2021

- You must work independently.
- Write your solutions clearly and show all of your work.
- Include your name, student ID number, and email address.
- Email a pdf file of your solution to `ugrad_services@math.utah.edu` by the deadline.
- A winner will be decided on the basis of the best solution submitted. If no best solution can be determined (i.e. there exist relatively identical solutions), the winner will be the student who submitted their solution first.
- Each submission will be given 3 points for a fully correct solution and 1-2 points for a partially correct solution. The winner of each problem set will get a bonus of ϵ points.
- Please don't just search online for a solution - that isn't the point of this contest.
- Enjoy the problems!

Problem 1 [1 point]: Prove that no set of nine consecutive integers can be partitioned into two sets with the product of the elements in the first set equal to the product of the elements in the second set.

Problem 2 [1 point]: Prove that any positive integer can be represented as

$$\pm 1 \pm 2^2 \pm 3^2 \pm \cdots \pm n^2$$

for some choice of n and some choice of the signs.

Problem 3 [1 point]: Let X be a subset of the positive integers closed under addition so the sum of any two not necessarily distinct elements in X is again in X. Suppose that $\{a_1, a_2, \ldots, a_n\}$ is the set of all positive integers not in X. Prove that

$$a_1 + a_2 + \cdots + a_n \leq n^2$$