PROBLEM 2: SQUARE PROBABILITIES

UNDERGRADUATE PROBLEM SOLVING CONTEST

Due Friday, November 13, 2009 by 5:00 PM

This problem has two parts. A correct answer to either part will be considered to be a partially correct solution, while correct answers to both parts will be necessary for a fully correct solution.

(a) A point is chosen at random from the interior of a square with vertices at (0, 0), (1, 0), (1, 1) and (0, 1). A line is drawn through this point and the point $(\frac{1}{2}, \frac{1}{4})$. What is the probability that the line intersects two adjacent sides of the square?

(b) Two points are chosen at random (independently) from the interior of the square. What is the probability that the line drawn through both points intersects two adjacent sides of the square?

In the spirit of the UPSC, you should not search the internet or look the solution up in a book.