EXERCISES I: MULTIPLYING LIKE RABBITS

Suppose a newly-born pair of rabbits (one male and one female) are put in a field. Rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pairs of rabbits. suppose our rabbits never die and that a female always produces one new pair (one male and one female) every month from the second month on. How many pairs will there be in one year?

The Lucas numbers $L_0, L_1, L_2, \ldots$ are a sequence that satisfy the same recursion relation as the Fibonacci numbers, namely

$$L_{n+2} = L_n + L_{n+1} \text{ for } n \geq 0$$

but different initial conditions:

$$L_0 = 2$$
$$L_1 = 1$$

Compute the first 10 Lucas numbers.

EXERCISES III: FIBONACCI 2-SUBSEQUENCES

The first few even terms in the Fibonacci sequence are

$$0, 1, 3, 8, 21, 55, 144, 377, \ldots .$$

Can you find a recursion relation that they satisfy?

The first few odd terms in the Fibonacci sequence are

$$1, 2, 5, 13, 34, 89, 233, 610, \ldots .$$

Can you find a recursion formula that they satisfy?

EXERCISES IV: FIBONACCI 3- AND 4-SUBSEQUENCES

The sequence of every third Fibonacci number starts

$$0, 2, 8, 34, 144, 610, \ldots .$$

Can you find a recursion formula for this sequence? Do the same for the sequence of every fourth Fibonacci number.

Show that
$$F_{n+r} = L_r F_n - (-1)^r F_{n-r},$$
where L_r is the rth Lucas number defined above.

Show that the Lucas numbers satisfy the same relation as the Fibonacci r-sequence on the previous page,

$$L_{n+r} = L_r L_n - (-1)^r L_{n-r}.$$

Notice if $n = r$, we get a nifty formula,

$$L_{2n} = L_n^2 - (-1)^n 2.$$

EXERCISES VI: BACK TO DETERMINANTS

Set
$$A = \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix} \qquad B = \begin{pmatrix} 6 & 4 \\ 2 & 8 \end{pmatrix} \qquad C = \begin{pmatrix} -1 & 0 \\ 3 & -2 \end{pmatrix}$$
Compute the following determinants:

(a) $\det(A)$

(b) $\det(B)$

(c) $\det(C)$

Compute the following products:

(a) AB

(b) BC

(c) AC

(d) CA

Compute the following determinants:

(a) $\det(AB)$

(b) $\det(BC)$

(c) $\det(AC)$

(d) $\det(CA)$

EXERCISES VII: TWO-BY-TWO FIBONACCI MATRICES

Set

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \quad .$$

(a) Compute A^2

(b) Compute A^3

(c) Compute A^4

(d) Compute A^5

(e) Compute A^n for any n.

(f) Compute $\det(A^n)$ for any n in two different ways.