1. ABC, 123

A student finds herself bored during math class. She starts to doodle and happens to notice that $123 \times 7 \times 11 \times 13 = 123123$. Pursuing her hunch, she next finds that $987 \times 7 \times 11 \times 13 = 987987$. She then is led to guess that if abc is any three digit number, then

$$abc \times 7 \times 11 \times 13 = abcabc.$$

Is her guess correct? Either find a counterexample (i.e. a number abc such that $abc \times 7 \times 11 \times 13 \neq abcabc$) or prove the formula in general.

2. FINDING N (EMO)

The positive integers 30, 72, and N have the property that the product of any two of them is divisible by the third. What is the smallest possible value of N?

3. FRIENDS AND ENEMIES AND FRIENDS' ENEMIES

Anne, Betty, Carol, Deb, Elise, and Frita are classmates. Any two of them are either friends or enemies, and the friend of an enemy is also an enemy. In how many different ways can these classmates be friends or enemies?

Consider the following situation that arises for the winner of a game show. Behind each of five doors, the game show organizers flipped a fair coin. If it landed heads, they placed a prize of $17,500 behind the door; if it landed tails, they placed a goat. After completing this procedure, the organizers revealed a further piece of information: it transpired that at least two prizes are present behind the five doors. The contestant has already won $10,000, but is now given the option of trading his $10,000 for the chance to pick one of the doors (and win what lies behind it). Determine what he should do by calculating the expected payoff of choosing a door.