MATH CIRCLE FINALE

May 4, 2005

1. With $1000 a rancher is to buy steers at $25 each and cows at $26 each. If he has no money left over, and he bought at least one animal of each kind, how many cows did he buy?

2. How many triples of integers greater than or equal to zero satisfy the simultaneous equations

$$ab + bc = 44$$
$$ac + bc = 23 \ ?$$

3. Find a degree three polynomial $P(x)$ so that $xP(x-1) = (x-3)P(x)$.

4. How many integers from 1 to 2005 have base three representations that do not contain the digit 2?

5. What is the smallest integer multiple of 49 whose digits are all the same?