

MATH CIRCLE CONTEST

March 9, 2005

A biologist is studying a slow-growing bacteria population. Based on empirical evidence, she finds that it is reasonable to model the growth in the population at time $n + 1$ from time n as

$$\ln \left(1 + \frac{1}{n+1} \right).$$

She needs a formula for $p(n)$, the population at time n . Find such a formula given that the initial population $p(0)$ is 100.

2.

Prove or disprove: there exists a one-to-one onto map from the set of integers none of whose digits is a 9 to the set of integers all of whose digits are 0's and 1's.

3.

Consider the familiar Towers of Hanoi problem with the following wrinkle: each disc may only be moved to an adjacent tower. (Moving a disc from the first to third tower is thus no longer allowed.) Establish a formula for the minimal number of moves needed to move n discs of increasing size from the first tower to the fourth. Prove that your formula indeed gives the minimal number necessary.

4.

How many nonnegative integer solutions are there to the inequality

$$x_1 + x_2 + x_3 + x_4 \leq 500?$$