

MATH CIRCLE CONTEST III
February 9, 2005

1. Evaluate

$$\frac{1}{2 \cdot 4} + \frac{1}{4 \cdot 6} + \frac{1}{6 \cdot 8} + \cdots + \frac{1}{18 \cdot 20}.$$

Hint: notice that

$$\frac{1}{n(n+2)} = \frac{1}{2n} - \frac{1}{2(n+2)}.$$

2. Let r , s , and t be the roots of $x^3 - 6x^2 + 5x - 7 = 0$. Find

$$\frac{1}{r^2st} + \frac{1}{rs^2t} + \frac{1}{rst^2}.$$

3. Given a function $f(x)$ satisfying

$$f(x) + 2f(1/(1-x)) = x,$$

find $f(2)$.

4. Assume

$$(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + \cdots + a_{2n}x^{2n}$$

is an identity in x . Find

$$a_0 + a_2 + a_4 + \cdots + a_{2n}$$

in terms of n .

5. Find x^2 if $\sqrt[3]{x+9} - \sqrt[3]{x-9} = 3$. (Hint: x^2 is an integer.)

6. Find

$$\sum_{k=1}^{11} c_k^2,$$

where

$$c_n = n + \frac{1}{2n + \frac{1}{2n + \dots}}.$$