1. SQUARES WITH MAGICAL PROPERTIES
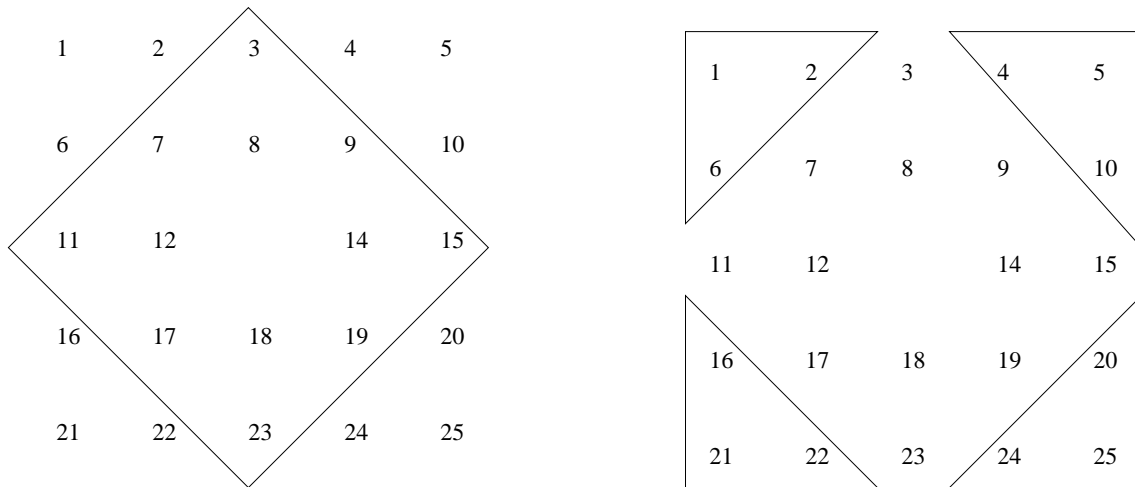
Consider the following two pictures.

The sum of the numbers in the diamond in the left figure is 156. (Note that the middle element is excluded from the diamond.) The sum of the numbers in the corner of the right figure is also 156.

Now consider two 21-by-21 squares with entries $1, \dots, 441$ arranged as above. In one square, consider the sum of the numbers in the diamond (again excluding the middle entry). In the other square consider the sum of the corners. Prove that these two sums are the same. What is their common value?

2. Produce Pyramids

The head grocer at Smith's decides to get creative and build a massive pyramid of oranges as the centerpiece of his produce display. The top of the pyramid consists of a single orange. The second layer consists of three oranges,
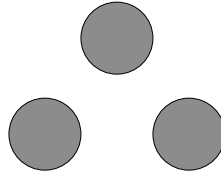
The third layer looks like
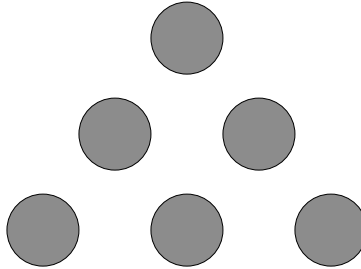
And so on. The grocer wants a pyramid with 100 layers of oranges. How many oranges does such a pyramid require?

3. YOU CAN DO IT WITHOUT A CALCULATOR

(a) Compute the remainder of $2^{100} + 2^{99} + \cdots + 2^1 + 2^0$ when divided by 3.

(b) Compute the remainder of 100! when divided by 101.