MATH CIRCLE SPRING CONTEST II
April 2, 2008

1. WARM-UP

Show that 10^{10} cannot be written as a product of two natural numbers which do not contain 0 in their decimal representation.

2. TOASTY

Point E is selected on side AB of $\triangle ABC$ in such a way that $AE : EB = 1 : 3$ and point D is selected on side BC so that $CD : DB = 1 : 2$. The point of intersection of AD and CE is F. Find

$$\frac{EF}{FC} + \frac{AF}{FD}.$$

3. it's getting hot in here

In $\triangle ABC$, segments are drawn from A to the trisection points of side BC. A median drawn from B is divided, by these segments, in the continued ratio $x : y : z$. If $x \leq y \leq z$ then find $x : y : z$. See Figure 1.

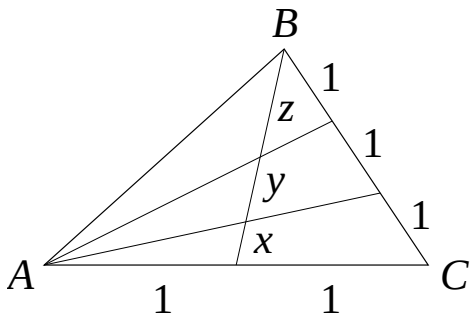

Figure 1

4. HOT ENOUGH FOR YOU?

In $\triangle ABC$, E is the midpoint of side AC and D is a point on side BC such that $2(BD) = DC$; AD and BE intersect at F. Find the ratio of the area of $\triangle BDF$ to the area of quadrilateral $FDCE$. See Figure 2.

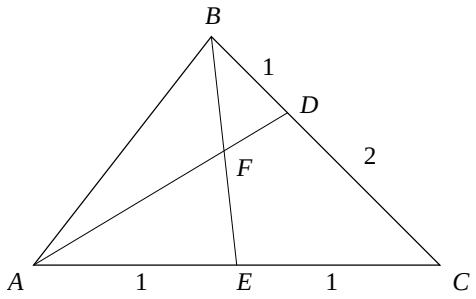

FIGURE 2