

MATH CIRCLE CONTEST II  
December 5, 2007

1.

Find the remainder of  
after dividing by 18.

$$1! + 3! + 5! + \cdots + 49!$$

Balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row of two balls, three in third row and so on. If 669 more balls are added then all the balls can be arranged in the shape of a square and each of sides then contains 8 balls less than the side of the triangle had. Determine the initial number of balls.

3.

Mary the mathematician want to define an associative multiplication on the four element set  $S = \{e, a, b, c\}$  which satisfies

- (i) The multiplication is not commutative. That is, there is at least one pair of element  $x$  and  $y$  in  $S$  such that  $xy \neq yx$ .
- (ii)  $e$  behaves like an identity:  $ex = xe = x$  for all  $x$  in  $S$ .
- (iii) Inverses exist: for each  $x$  in  $S$  there is some element (say  $x^{-1}$ ) so that  $xx^{-1} = x^{-1}x = e$ .

Can Mary do it?

4.

The digits of a number are 1,2,3,4,5,6,7,8,9 written in any order. Of all the numbers that can be so formed (that is, all the nine digit numbers where each of the integers 1 through to 9 must appear without repetition) how many are divisible by 11?

5.

A positive integer  $N$ , when divided by 10, 9, 8, 7, 6, 5, 4, 3, and 2, leaves remainders 9, 8, 7, 6, 5, 4, 3, 2 and 1 respectively. Find the least value of  $N$ . (Hint: What happens when you add one to the number?)