1(A). TILING, PART ONE

Let T_n denote the number of ways to tile a rectangle of size 3-by-n with 1-by-3 tiles. For instance, $T_4 = 3$ since we have the following three tilings of the 3-by-4 rectangle

Find T_{10}.

1(B) TILING, PART 2

Using the notation of the previous problem, prove that

$$T_{3n} = 1 + T_2 + T_5 + T_8 + \cdots + T_{3n-1}.$$

For $n \geq 3$, prove

$$T_{2n+1} = T_n^2 + T_{n-1}^2 + 2T_n T_{n-2}.$$

2.

Let r denote the radius of a circle inscribed in an acute triangle with perimeter P and area A. Show that
$$A = \frac{Pr}{2}.$$

3.

Consider a circle centered at a point P. Suppose AB and AC are two chords the circle meeting at the point A. Prove that the angle BAC is half of the angle subtended by the arc passing through B and C (but not A).