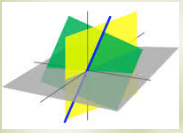
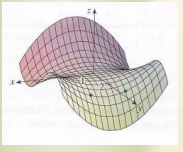


$$f_x = \frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$f_y = \frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x,y+h) - f(x,y)}{h}$$

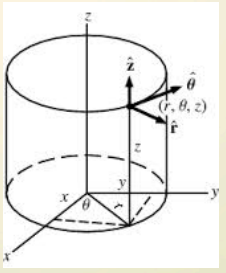


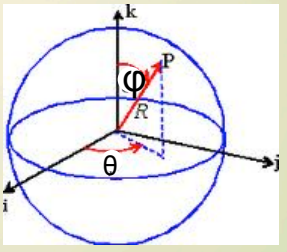
$$\int_0^1 \int_0^{2y} xy\,dx\,dy = \int_0^1 \left[\frac{x^2}{2} y\right]_{x=0}^{x=2y} dy$$

$$= \int_0^1 \frac{(2y)^2}{2} y\,dy = \int_0^1 2y^3\,dy$$

$$= \left[\frac{y^4}{2}\right]_{y=0}^{y=1} = \frac{1}{2}$$

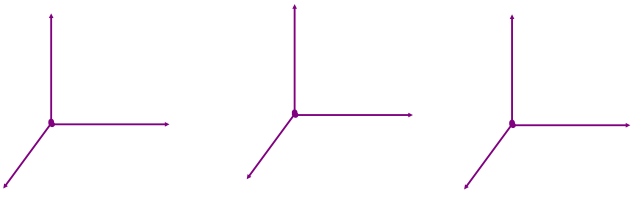
Cylindrical and Spherical Coordinates





We can describe a point, *P,* in three different ways.

Cartesian Cylindrical Spherical



Cylindrical Coordinates

$x = r\cos\theta$ $\quad r = \sqrt{x^2 + y^2}$

$y = r\sin\theta$ $\quad \tan\theta = y/x$

$z = z$ $\quad z = z$

Spherical Coordinates

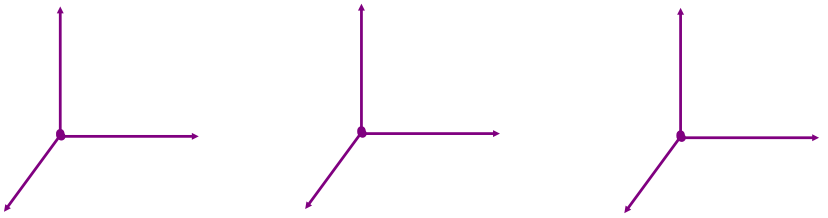
$x = \rho\sin\varphi\cos\theta$ $\quad \rho = \sqrt{x^2 + y^2 + z^2}$

$y = \rho\sin\varphi\sin\theta$ $\quad \tan\theta = y/x$

$z = \rho\cos\varphi$ $\quad \cos\varphi = \dfrac{z}{\sqrt{x^2 + y^2 + z^2}}$

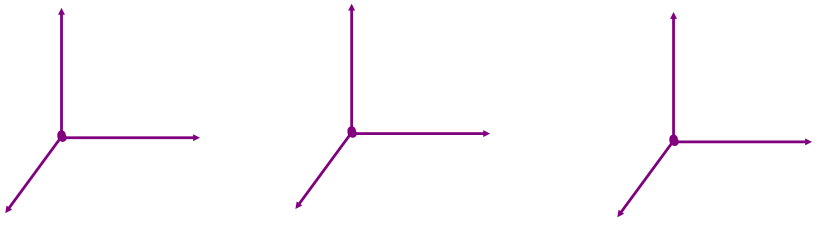
Easy Surfaces in Cylindrical Coordinates

a) $r = 1$ b) $\theta = \pi/3$ c) $z = 4$



Easy Surfaces in Spherical Coordinates

a) $\rho = 1$ b) $\theta = \pi/3$ c) $\varphi = \pi/4$



EX 1 Convert the coordinates as indicated

a) $(3, \pi/3, -4)$ from cylindrical to Cartesian.

b) $(-2, 2, 3)$ from Cartesian to cylindrical.

EX 2 Convert the coordinates as indicated

a) $(8, \pi/4, \pi/6)$ from spherical to Cartesian.

b) $(2\sqrt{3}, 6, -4)$ from Cartesian to spherical.

EX 3 Convert from cylindrical to spherical coordinates.

$(1, \pi/2, 1)$

EX 4 Make the required change in the given equation.

a) $x^2 - y^2 = 25$ to cylindrical coordinates.

b) $x^2 + y^2 - z^2 = 1$ to spherical coordinates.

c) $\rho = 2\cos\varphi$ to cylindrical coordinates.

EX 4 Make the required change in the given equation (continued).

d) $x + y + z = 1$ to spherical coordinates.

e) $r = 2\sin\theta$ to Cartesian coordinates.

f) $\rho\sin\theta = 1$ to Cartesian coordiantes.