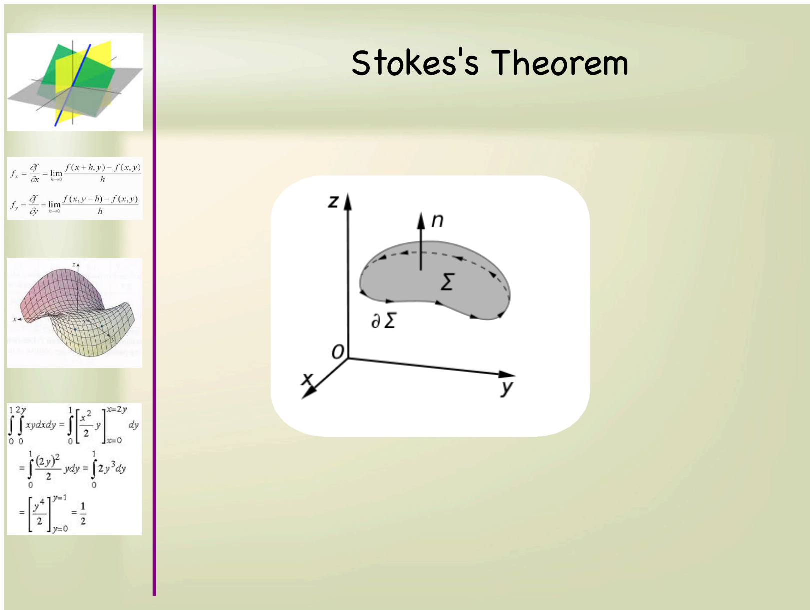


Remember this form of Green's Theorem:

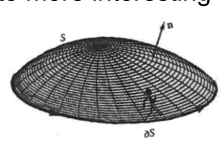
$$\oint_C \vec{F}\cdot d\vec{r} = \iint_R \nabla\times\vec{F}\cdot\hat{k}\, dA$$

where $\vec{F}(x,y) = M(x,y)\hat{\imath} + N(x,y)\hat{\jmath},$

C is a simple closed positively-oriented curve that encloses a closed region, R, in the xy-plane.

It measures circulation along the boundary curve, C.

Stokes's Theorem generalizes this theorem to more interesting surfaces.



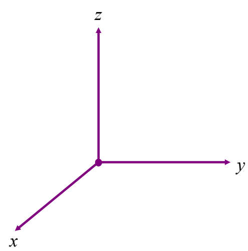
Stokes's Theorem

For $\vec{F}(x,y,z) = M(x,y,z)\hat{\imath}+N(x,y,z)\hat{\jmath}+P(x,y,z)\hat{k},$

M, N, P have continuous first-order partial derivatives.
S is a 2-sided surface with continuously varying unit normal, $\hat{n}$,
C is a piece-wise smooth, simple closed curve, positively-oriented that is the boundary of S,
$\hat{T}$ is the unit tangent vector to C,

then $$\oint_C \vec{F}\cdot\hat{T}\, ds = \iint_S (\nabla\times\vec{F})\cdot\hat{n}\, dS$$

EX 1 Verify Stokes's Theorem for $\vec{F} = y^2\hat{\imath} - x\hat{\jmath} + 5z\hat{k}$ if
S is the paraboloid $z = x^2 + y^2$
with the circle $x^2 + y^2 = 1$ as its boundary.



EX 2 Use Stokes's Theorem to calculate $\iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dS$
for $\vec{F} = xz^2\hat{\imath} + x^3\hat{\jmath} + \cos(xz)\hat{k}$
where S is the part of the ellipsoid $x^2 + y^2 + 3z^2 = 1$
below the xy-plane and $\hat{n}$ is the lower normal.

EX 3 Let S be a solid sphere. Show that $\iint_S (\nabla \times \vec{F}) \cdot \hat{n} \, dS = 0$

a) by using Stokes's Theorem

b) by using Gauss's Theorem