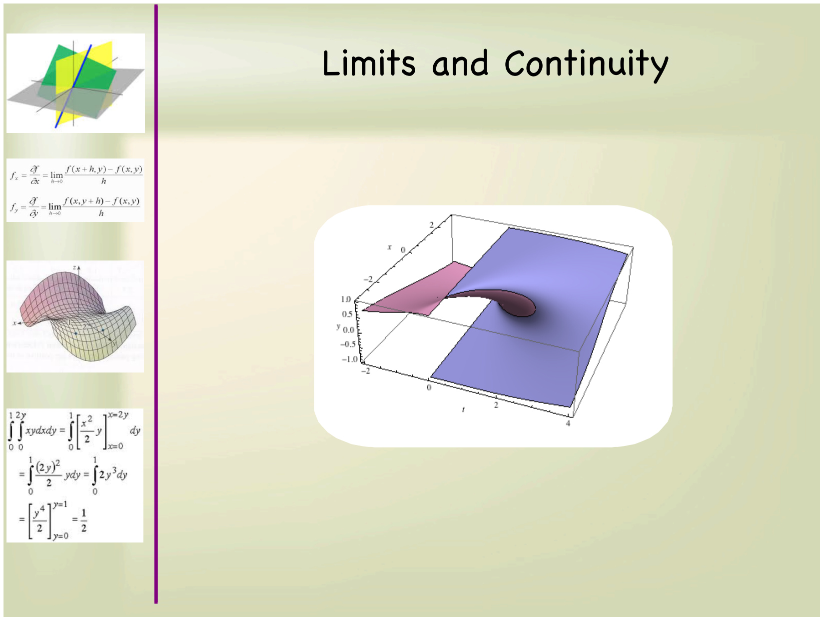


Review from Calculus 1

Definition: Continuity at a Point

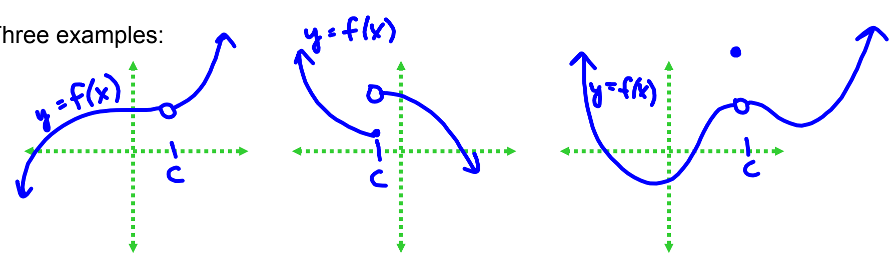
Let *f* be defined on an open interval containing *c*. We say that *f* is continuous at *c* if

$$\lim_{x \to c} f(x) = f(c)$$

This indicates three things:

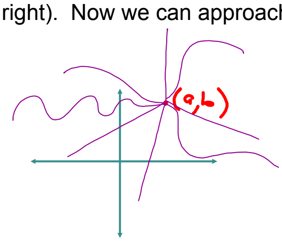
1. The function is defined at $x = c$.
2. The limit exists at $x = c$.
3. The limit at $x = c$ needs to be exactly the value of the function at x = c.

Three examples:



Limits and Continuity

Intuitively, $\lim_{(x,y)\to(a,b)} f(x,y)=L$ means that as the point *(x,y)* gets very close to *(a,b)*, then *f(x,y)* gets very close to L. When we did this for functions of one variable, it could approach from only two sides or directions (left or right). Now we can approach *(a,b)* from infinitely many directions.



Definition of Limit

$\lim_{(x,y)\to(a,b)} f(x,y)=L$ means that for all $\varepsilon>0$ there exists a corresponding $\delta>0$ such that $|f(x,y)-L|<\varepsilon$ provided that $0<|(x,y)-(a,b)|<\delta$. We can make *f(x,y)* as close as we'd like to L by choosing *(x,y)* sufficiently close to *(a,b)*.

$$|(x,y)-(a,b)|=\sqrt{(x-a)^2+(y-b)^2}$$

EX 1 Find $\lim_{(x,y)\to(-2,1)}\left(xy^3-xy+3y^2\right)$.

EX 2 Find $\lim_{(x,y)\to(0,0)}\dfrac{1+xy}{\cos(xy)}$.

EX 3 $\lim_{(x,y)\to(0,0)} \frac{\tan(x^2+y^2)}{x^2+y^2}$

EX 4 Show that $\lim_{(x,y)\to(0,0)} \frac{xy+y^3}{x^2+y^2}$ does not exist.

EX 5 Find the limits.

a) $\lim_{(x,y)\to(0,0)} \frac{x^3}{x^2+y^2}$ *Hint: Use polar coordinates.*

b) $\lim_{(x,y)\to(0,0)} \frac{x}{x^2+y^2}$

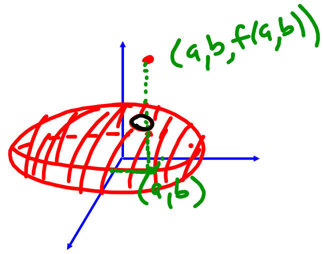
c) $\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$

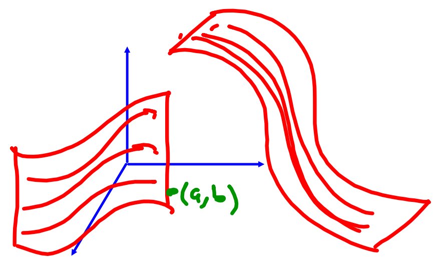
Continuity

A function $f(x,y)$ is continuous at (a,b) if $f(a,b) = \lim_{(x,y)\to(a,b)} f(x,y)$

This indicates three things:

a) the function is defined at (a,b),
b) the limit of f as $(x,y) \to (a,b)$ exists, and
c) the limit of f at (a,b) is exactly the same as $f(a,b)$.





Composition of Functions

If a function, g, of two variables is continuous at (a,b) and a function, f, of one variable is continuous at $g(a,b)$, then

$(f \circ g)(x,y) = f(g(x,y))$ is continuous at (a,b).

EX 6 Show that $f(x,y) = \sin(x^2-4xy)$ is continuous everywhere.

EX 7 Determine where $f(x,y) = \ln(1-x^2-y^2)$ is continuous.

EX 8 Is $f(x,y) = \begin{cases} \dfrac{\sin(xy)}{xy} & if\ xy \neq 0 \\ 1 & if\ xy = 0 \end{cases}$ continuous everywhere?