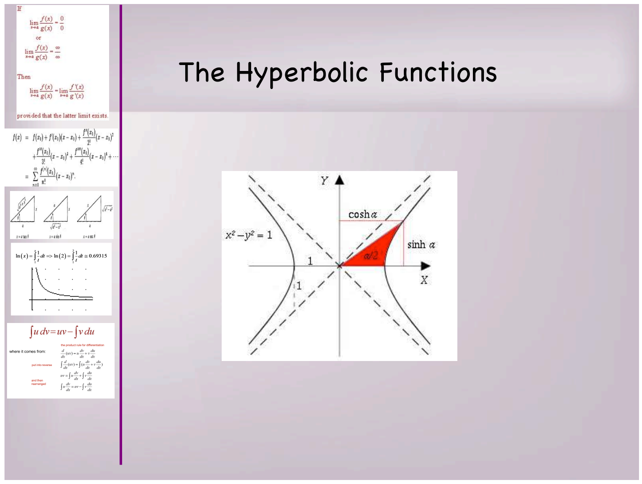


Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$ (called "sinsh")

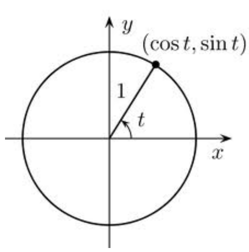
$$\cosh x = \frac{e^x + e^{-x}}{2}$$

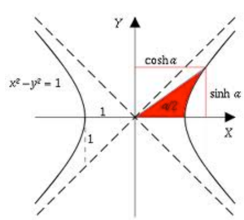
$$\tanh x = \frac{\sinh x}{\cosh x}$$

Relation to the trigonometric functions:

1. (cos θ, sin θ) is a point on the unit circle.
2. y= sin x is an odd function.
3. y = cos x is an even function.
4. $\sin^2\theta + \cos^2\theta = 1$

1. (cosh θ, sinh θ) is a point on the unit hyperbola.
2. y = sinh x is an odd function.
3. y = cosh x is an even function.
4. $\cosh^2\theta - \sinh^2\theta = 1$





Prove $\cosh^2\theta - \sinh^2\theta = 1$.

$D_x(\sinh x) = \cosh x$	$D_x(\cosh x) = \sinh x$
$D_x(\tanh x) = \text{sech}^2 x$	$D_x(\coth x) = -\text{csch}^2 x$
$D_x(\text{sech } x) = -\text{sech } x \tanh x$	$D_x(\text{csch } x) = -\text{csch } x \coth x$

EX 1 $D_x(\coth(4x) \sinh x) =$

EX 2 $\int \tanh x \ln(\cosh x)\, dx =$

EX 3 Verify this identity.

$$\tanh(x-y)=\frac{\tanh x-\tanh y}{1-\tanh x\tanh y}$$

Hint: $\tanh x=\frac{e^x-e^{-x}}{e^x+e^{-x}}$

Inverse Hyperbolic Functions

Let $y = \sinh x$ <=> $x = \sinh^{-1} y$ (if the inverse exists).

Find $x = \sinh^{-1} y$.

$$\sinh^{-1} x = \ln(x + \sqrt{x^2+1}) \qquad \cosh^{-1} x = \ln(x + \sqrt{x^2-1}) \quad x \geq 1$$

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \quad x \in (-1,1) \qquad \operatorname{sech}^{-1} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) \quad x \in (0,1]$$

How do we find the derivative of these functions?

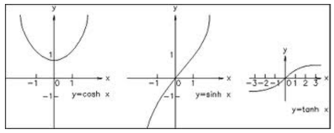
$$D_x(\sinh^{-1} x) = D_x\left(\ln\left(x + \sqrt{x^2+1}\right)\right)$$

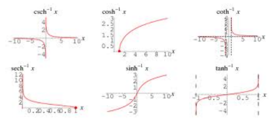
$$D_x\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{x^2+1}} \qquad D_x\left(\cosh^{-1} x\right) = \frac{1}{\sqrt{x^2-1}} \quad x > 1$$

$$D_x\left(\tanh^{-1} x\right) = \frac{1}{1-x^2} \quad x \in (-1,1) \qquad D_x\left(\operatorname{sech}^{-1} x\right) = \frac{-1}{x\sqrt{1-x^2}} \quad x \in (0,1)$$

EX 4 Find y' .

$$y = x^2 \sinh^{-1}(x^5)$$


y=cosh x
y=sinh x
y=tanh x


csch⁻¹ x
cosh⁻¹ x
coth⁻¹ x
sech⁻¹ x
sinh⁻¹ x
tanh⁻¹ x