

The Natural Exponential Function

if $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \frac{0}{0}$
or
 $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$

Then $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)}$

provided that the latter limit exists.

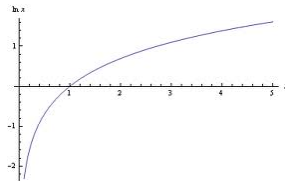
$f(x) = f(x) + f(x)(x-a) + \frac{f''(x)}{2!}(x-a)^2 + \dots$

$\frac{f'(x)}{g'(x)} = \frac{f'(x)}{g'(x)} + \frac{f''(x)}{2!g''(x)}(x-a) + \dots$

$\int u dv = uv - \int v du$

The Natural Exponential Function

Remember the graph of $y = \ln x$.



It is strictly monotonic, so it has an inverse function.

Draw it.

Domain:

Range:

Let's call the inverse function "exp."

$$\ln(\exp(x)) =$$

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Definition: Let e be a real number such that $\ln e = 1$.

$$r \in \mathbb{R}, \exp(r) = \exp(r \ln e) \quad \text{since } \ln e = 1$$

Theorem Let a and b be real numbers. Then $e^a e^b = e^{a+b}$ and $\frac{e^a}{e^b} = e^{a-b}$

Proof:

How do we take a derivative?

Let $y = e^x \Leftrightarrow \ln y = x$

EX 1 Find y' .

$$y = e^{x^2-3x}$$

EX 2 Find y' . $y = e^{\sqrt{x} \ln x}$

EX 3 For $f(x)$ analyze the graph. (I.e. min, max, concavity, inflection pts, sketch.)

$$f(x) = e^x - e^{-x}$$

Since $D_x[e^x] = e^x$, then $\int e^x dx = e^x + C$.

EX 4 Evaluate these integrals.

a) $\int e^{-6x} dx$

b) $\int e^{(x+e^x)} dx$

c) $\int \frac{e^{3/x}}{x^2} dx$