

# Graphs of Polar Equations

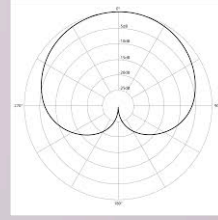
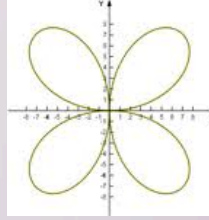
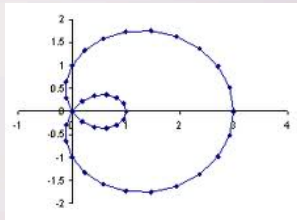
If  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$   
 or  
 $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$   
 Then  
 $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$   
 provided that the latter limit exists.

$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$   
 $= \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$

$\ln(x) = \int_1^x \frac{1}{t} dt \Rightarrow \ln(2) = \int_1^2 \frac{1}{t} dt \approx 0.69315$

$\int u dv = uv - \int v du$

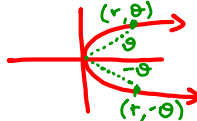
where it comes from:  
 product rule for differentiation:  $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$   
 put into reverse:  $\int \frac{d}{dx}(uv) = \int u \frac{dv}{dx} + \int v \frac{du}{dx}$   
 and then rearrange:  $\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$



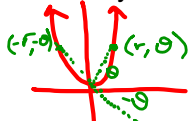
## Graphs of Polar Equations

### Symmetry

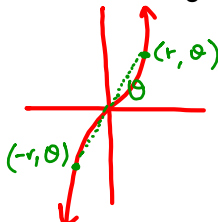
About the x-axis  $\Rightarrow$  replacing  $(r, \theta)$  with  $(r, -\theta)$  produces equivalent equation



About the y-axis  $\Rightarrow$  replacing  $(r, \theta)$  with  $(-r, -\theta)$  produces equivalent equation

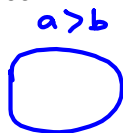


About the origin  $\Rightarrow$  replacing  $(r, \theta)$  with  $(-r, \theta)$  produces equivalent equation

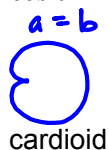


## Polar Equations

limaçon



$$r = a \pm b \cos \theta$$



$$r = a \pm b \sin \theta$$



lemniscate

$$r^2 = \pm a \cos(2\theta)$$



$$r^2 = \pm a \sin(2\theta)$$

rose

$$r = a \cos(n\theta)$$

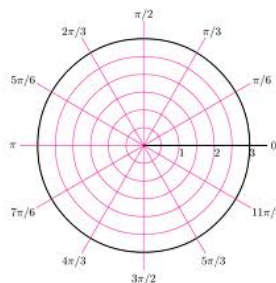
$$r = a \sin(n\theta)$$



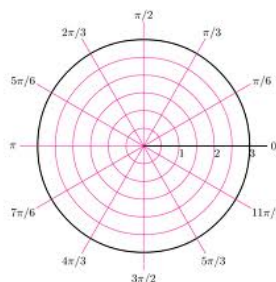
( $n$  leaves if  $n$  odd)  
 ( $2n$  leaves if  $n$  even)

EX 1 Sketch a graph of the given polar equations.

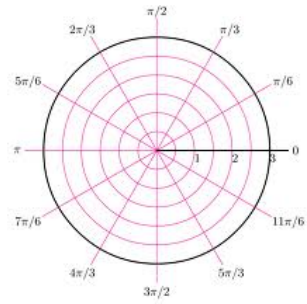
a)  $r = 4 \sin \theta$



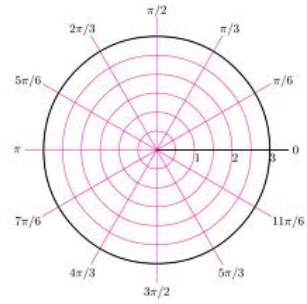
b)  $r = -16 \cos(2\theta)$



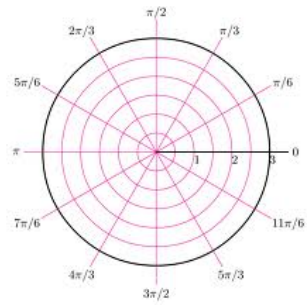
c)  $r = 4 - 3\sin \theta$



d)  $r = 2\theta$



e)  $r = \sqrt{2} - \sqrt{2} \sin \theta$



f)  $r^2 = 4 \cos(2\theta)$

