If

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$$

or

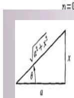
$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$$

Then

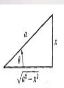
$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

provided that the latter limit exists.

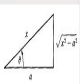
$$\begin{aligned} f(x) &= f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 \\ &\quad + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \frac{f''''(x_0)}{4!}(x - x_0)^4 + \cdots \\ &= \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n. \end{aligned}$$



$x = a\tan\theta$

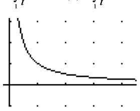


$x = a\sin\theta$



$x = a\sec\theta$

$$\ln(x) = \int_1^x \frac{1}{t}\,dt \Rightarrow \ln(2) = \int_1^2 \frac{1}{t}\,dt \cong 0.69315$$



$$\int u\,dv = uv - \int v\,du$$

where it comes from: the product rule for differentiation

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

put into reverse

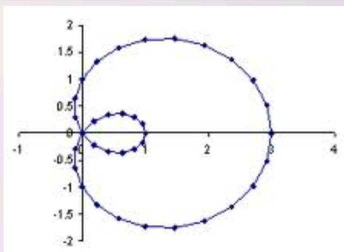
$$\int \frac{d}{dx}(uv) = \int \left(u\frac{dv}{dx} + v\frac{du}{dx}\right)$$

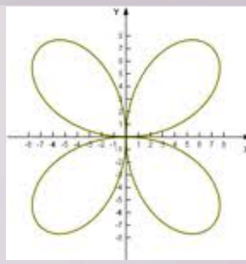
$$uv = \int u\frac{dv}{dx} + \int v\frac{du}{dx}$$

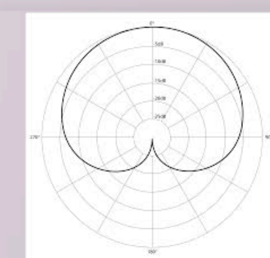
and then rearranged

$$\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$$

Graphs of Polar Equations



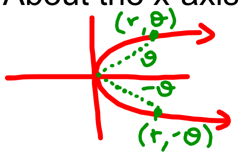




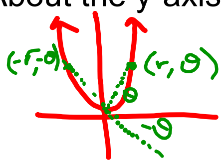
Graphs of Polar Equations

Symmetry

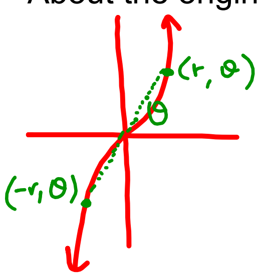
About the x-axis ⇒ replacing (r,θ) with $(r,-\theta)$ produces equivalent equation



About the y-axis ⇒ replacing (r,θ) with $(-r,-\theta)$ produces equivalent equation



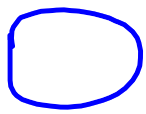
About the origin ⇒ replacing (r,θ) with $(-r,\theta)$ produces equivalent equation



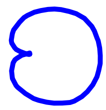
Polar Equations

limacon $r = a \pm b\cos\theta$ $r = a \pm b\sin\theta$

$a > b$

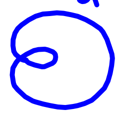


$a = b$

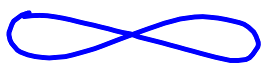


cardioid

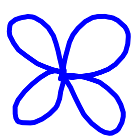
$a < b$



lemniscate $r^2 = \pm a\cos(2\theta)$ $r^2 = \pm a\sin(2\theta)$



rose $r = a\cos(n\theta)$ $r = a\sin(n\theta)$



(n leaves if n odd
2n leaves if n even)

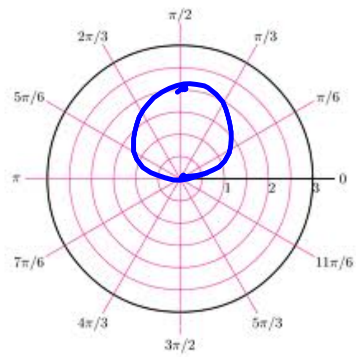
EX 1 Sketch a graph of the given polar equations.

a) $r = 4 \sin \theta$

circle.

(goes through origin)

r	θ
4	$\pi/2$
0	0

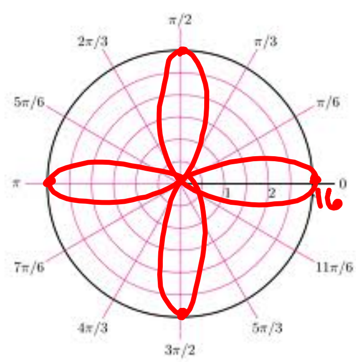


b) $r = -16 \cos(2\theta)$

(rose) $n = 2$

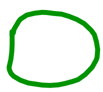
$\Rightarrow$ 4 leaves (petals)

r	θ
-16	0
16	$\pi/2$
-16	π

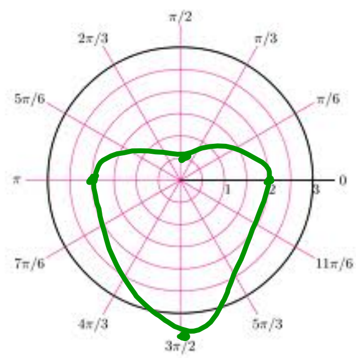


c) $r = 4 - 3\sin\theta$

limacon $a=4,\ b=3$
$a>b$

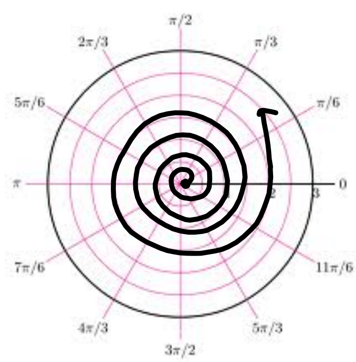


r	θ
1	$\pi/2$
7	$3\pi/2$
4	0

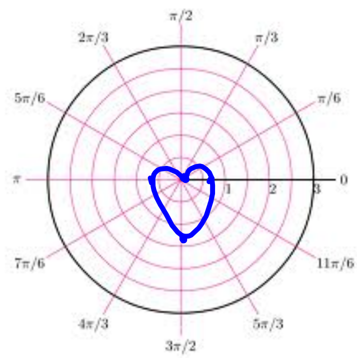


d) $r = 2\theta$

spiral



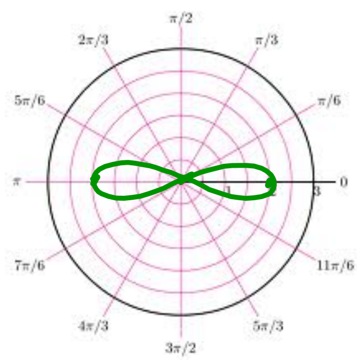
e) $r = \sqrt{2} - \sqrt{2}\,\sin\theta$



Cardioid

r	θ
0	$\pi/2$
$2\sqrt{2}$	$3\pi/2$
$\sqrt{2}$	$0, \pi$

f) $r^2 = 4\cos(2\theta)$



lemniscate

r	θ
± 2	0
	$\pi/2$
0	$\pi/4$

$r^2 = -4$

Conclusion:

Intro into graphing some polar eqns.