

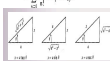
## Improper Integrals With Indefinite Integrands

If  $\lim_{x \rightarrow a^+} \frac{f(x)}{g(x)} = \frac{0}{0}$   
or  
 $\lim_{x \rightarrow a^+} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$

Then  $\lim_{x \rightarrow a^+} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a^+} \frac{f'(x)}{g'(x)}$

provided that the latter limit exists.

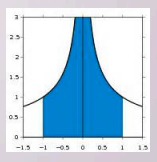
$f(x) = f_1(x) + f_2(x) + \dots + f_n(x) - x^2$   
 $\frac{f(x)}{x^2} = \frac{f_1(x)}{x^2} + \frac{f_2(x)}{x^2} + \dots + \frac{f_n(x)}{x^2} - 1$   
 $\lim_{x \rightarrow a^+} \frac{f(x)}{x^2} = \lim_{x \rightarrow a^+} \frac{f_1(x)}{x^2} + \lim_{x \rightarrow a^+} \frac{f_2(x)}{x^2} + \dots + \lim_{x \rightarrow a^+} \frac{f_n(x)}{x^2} - 1$



$\lim_{x \rightarrow a^+} \frac{f(x)}{x^2} = \lim_{x \rightarrow a^+} \frac{f_1(x)}{x^2} + \lim_{x \rightarrow a^+} \frac{f_2(x)}{x^2} + \dots + \lim_{x \rightarrow a^+} \frac{f_n(x)}{x^2} - 1$

$\int u dv = uv - \int v du$

Example:  $\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$



**Improper Integrals: Infinite Integrands**

Look at  $\int_{-1}^2 \frac{1}{x^4} dx$ . Can we just do the integral?

**Definition**

Let  $f(x)$  be continuous on  $[a, b]$  and

$$\lim_{x \rightarrow b^-} |f(x)| = \infty \Rightarrow \int_a^b f(x) dx = \lim_{t \rightarrow b^-} \int_a^t f(x) dx$$

if the limit exists and is finite, otherwise it diverges.

$$\text{EX 1} \quad \int_1^3 \frac{dx}{(x-1)^{4/3}}$$

$$\text{EX 2} \quad \int_0^9 \frac{dx}{\sqrt{9-x}}$$

$$\text{EX 3} \quad \int_0^1 \frac{1}{x^p} dx, \quad p \geq 1$$

Definition

If  $f$  is continuous on  $[a, b]$  except at  $x=c$  where  $a < b < c$

and  $\lim_{x \rightarrow c} |f(x)| = \infty$

then  $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$

if both integrals converge. Otherwise it diverges.

EX 4  $\int_{-5}^0 \frac{1}{(x+3)^2} dx$

EX 5  $\int_{-3}^1 \frac{5}{(x+2)^{3/5}} dx$