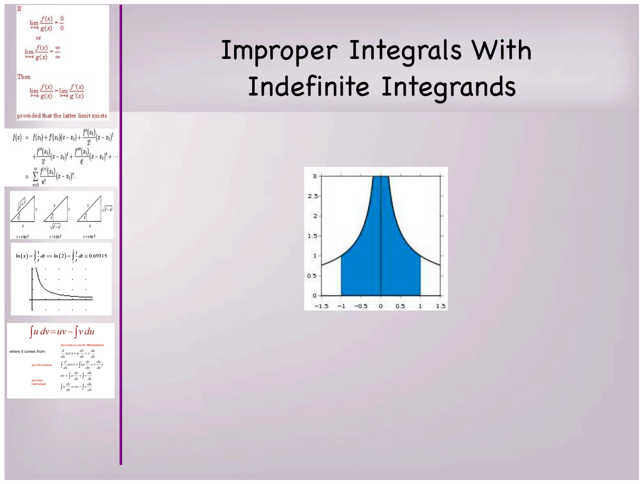

Improper Integrals: Infinite Integrands

Look at $\int_{-1}^{2} \frac{1}{x^4}\,dx$. Can we just do the integral?

Definition

Let f(x) be continuous on [a,b] and

$$\lim_{x \to b^-} |f(x)| = \infty \Rightarrow \int_a^b f(x)dx = \lim_{t \to b^-} \int_a^t f(x)dx$$

if the limit exists and is finite, otherwise it diverges.

EX 1 $\int_1^3 \frac{dx}{(x-1)^{4/3}}$

EX 2 $\int_0^9 \frac{dx}{\sqrt{9-x}}$

EX 3 $\int_0^1 \frac{1}{x^p}\,dx, \quad p \geq 1$

Definition

If f is continuous on $[a,b]$ except at $x=c$ where $a<b<c$

and $\lim_{x\to c} | f(x) |= \infty$

then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

if both integrals converge. Otherwise it diverges.

EX 4 $\int_{-5}^{0} \frac{1}{\left(x+3\right)^2} dx$

EX 5 $\int_{-3}^{1} \frac{5}{\left(x+2\right)^{3/5}} dx$