

## Indeterminate Forms of Type 0/0

Indeterminate forms

$\frac{0}{0}$

 $\frac{\infty}{\infty}, 0 \cdot \infty, \infty - \infty, 0^\infty, \infty^0, 1^\infty$

Previously we found the limit of an expression which appeared to approach  $\frac{0}{0}$ .

Determine this limit. 
$$\lim_{x \rightarrow 2} \frac{x^2 - 2x}{x^2 + 3x - 10}$$

We also were able to geometrically determine this limit. 
$$\lim_{x \rightarrow 0} \frac{\sin x}{x}$$

**L'Hopital's Rule:**

If  $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$  and  $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$  exists

(either finite or  $\pm\infty$ ), then  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$ .

This makes both of the previous problems more simple.

EX 1 Determine these limits using the rule above.

a)  $\lim_{x \rightarrow 2} \frac{x^2 - 2x}{x^2 + 3x - 10}$

b)  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$

EX 2  $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{2 \sin x}$

EX 3  $\lim_{x \rightarrow 0^+} \frac{7^{\sqrt{x}} - 1}{2^{\sqrt{x}} - 1}$

$$\text{EX 4} \quad \lim_{x \rightarrow 0} \frac{\sin x - \tan x}{x^2 \sin x}$$

$$\text{EX 5} \quad \lim_{x \rightarrow 0} \frac{\cos x}{x}$$

$$\text{EX 6} \quad \lim_{x \rightarrow 0^+} \frac{\int_0^x \sqrt{t} \cos t \, dt}{x^2}$$

$$\text{EX 7} \quad \lim_{x \rightarrow 0^+} \frac{\sin x + \tan x}{e^x + e^{-x} - 2}$$