

Strategies for Integrating

$$\int \frac{f(x)}{g(x)} dx$$

The collage on the left side of the slide contains the following content:

- L'Hopital's Rule:** If $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)}$ provided that the latter limit exists.
- Quotient Rule:** $f'(x) = f(x)g'(x) - f'(x)g(x)$
- Integration by Parts:** $\int u dv = uv - \int v du$
- Integration Table:**

$\int \frac{1}{x} dx = \ln x + C$	$\int \frac{1}{x^2} dx = -\frac{1}{x} + C$
$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$	$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln\left \frac{x-a}{x+a}\right + C$
$\int \frac{1}{x^2 + 2ax + a^2} dx = \frac{1}{x+a} + C$	$\int \frac{1}{x^2 - 2ax + a^2} dx = -\frac{1}{x-a} + C$

Strategies for Integration

1. u-substitution
2. integration by parts
3. trigonometric integrals (use identities)
4. rationalizing/trigonometric substitutions
5. partial fraction decomposition
6. integral tables
7. computer/calculator approximations

EX 1 $\int \frac{1}{t - \sqrt{2t}} dt$

$$\text{EX 2 } \int \frac{\sqrt{x^2 - 4x}}{x-2} dx$$

$$\text{EX 3 } \int \frac{\operatorname{sech}(\sqrt{x})}{\sqrt{x}} dx$$

EX 4 $\int x^2 \sqrt{25 - 4x^2} dx$