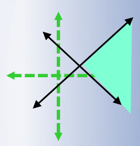
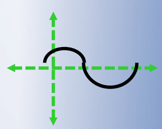
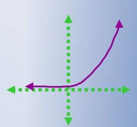
$5x-2y \le 75$



$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$



$S = Pe^{rt}$



$APY = (1+\frac{r}{n})^n - 1$

Math 1090 ~ Business Algebra

Section 5.3 Future Value of Annuities

Objectives:

- Determine the future value of an ordinary annuity.
- Solve problems involving annuities.

An annuity is a financial plan characterized by regular payments.

ex saving for retirement or college

Ordinary Annuity	Annuity Due
payments made at the end of each equal payment interval	payments made at the beginning of each equal payment interval

Ex 1: Suppose you invest $1000 at the end of each year for 5 years in an account that pays 10% interest compounded annually.

What is the value after 5 years?

compound interest

$S = P\left(1+\frac{r}{n}\right)^{nt}$

since our $n=1$

$\Rightarrow S = P(1+r)^t$

end of year 1: $1000

end of year 2: $1000(1+0.1)^1 + 1000$

end of year 3: $\underbrace{1000(1+0.1)^2}_{\text{from 1st yr deposit}} + \underbrace{1000(1+0.1)^1}_{\text{from 2nd yr deposit}} + \underbrace{1000}_{\text{from 3rd yr deposit}}$

end of year 4:

$1000(1+0.1)^3 + 1000(1+0.1)^2 + 1000(1+0.1)^1 + 1000$

end of year 5:

$1000(1+0.1)^4 + 1000(1+0.1)^3 + 1000(1+0.1)^2 + 1000(1+0.1)^1 + 1000$

notice at end of 5th yr, we have 5 terms; and it's sum of geom. sequence

$a_1 = 1000$, $d = (1+0.1)$, $n=5$

we know that formula:

$$\text{total balance} = \frac{1000\left(1-(1+0.1)^5\right)}{1-(1+0.1)}$$

Generally, for an ordinary annuity, the future value is

sum of geom seq

where $r_c = \frac{r}{n}$

R = monthly deposit

$N = nt$

$$S = \frac{R(1-(1+r_c)^N)}{\cancel{1}-(\cancel{1}+r_c)}$$

$$S = \frac{a_1(1-d^n)}{1-d}$$

$$S = \frac{R(1-(1+r_c)^N)}{-r_c}$$

compound interest formula $S = P(1+\underbrace{\tfrac{r}{n}}_{r_c})^{\underbrace{nt}_{N}}$

$$= \frac{R(-1+(1+r_c)^N)}{r_c}$$

$$\boxed{S = \frac{R((1+r_c)^N - 1)}{r_c}}$$

FV of ordinary annuity (we make regular deposits/payments to an acct; what is FV)

Ex 2: A story of twins

a) At the end of college, Thelma invests \$2000 at the end of each year for 8 years in an account that earns 10% compounded annually. After 8 years, she contributes nothing, but it continues to earn the same interest for 36 more years. How much does she have then?

$$S = \frac{R((1+r_c)^N - 1)}{r_c}$$

① ordinary FV (8 yrs)

$R = \$2000$

$r_c = \frac{0.1}{1} = 0.1$

$N = 1(8) = 8$

$$S = \frac{2000(1.1^8 - 1)}{0.1}$$

$S \simeq \$22,871.78$

(her total deposits: \$16,000)

② compound int.:

$S = 22,871.78(1+0.1)^{1(36)} \simeq \$707,027.91$

b) At the end of college Lewis invests nothing for 8 years. Then he puts \$2000 into an account at the end of each year for 36 years earning 10% interest compounded annually. How much does he have then?

$n = 1$

FV ordinary annuity:

$r_c = 0.1$

$$S = \frac{R((1+r_c)^N - 1)}{r_c}$$

$$S = \frac{2000(1.1^{36} - 1)}{0.1}$$

$\simeq \$598,253.61$

$N = 1(36) = 36$

(his total deposits: $2000(36) = \$72,000$)

Ex 3: How much should be invested quarterly (at the end of each quarter) at 12% interest compounded quarterly to pay off a debt of $30,000 in 6 years?

$$S = \frac{R((1+r_c)^N - 1)}{r_c}$$

$t = 6$ $\quad r_c = \frac{0.12}{4} = 0.03$

$r = 0.12$ $\quad N = 4(6) = 24$

$n = 4$

$S = 30{,}000$

$$30{,}000 = \frac{R((1+0.03)^{24} - 1)}{0.03}$$

$$\frac{30{,}000(0.03)}{(1.03^{24} - 1)} = R$$

$R \approx \$871.42$

(total deposits:
$871.42(24) \approx \$20{,}914.08$)

essentially same formula as FV ordinary annuity

Sinking Fund

$$R = S\left(\frac{r_c}{(1+r_c)^N - 1}\right)$$

The payment that needs to be invested every pay period to pay off debt of S at the end.

Ex 4: Find the future value of an account with \$100 deposited at the beginning of each month for 5 years into an account that pays 8% compounded monthly.

(deposits made at beginning of periods)

Future value of Annuity Due

$$S = R\left[\frac{(1+r_c)^{N+1}-1}{r_c}\right] - R$$

take away extra pymnt

looks like FV ordinary annuity w/ one extra payment

$$\boxed{S = R(1+r_c)\left[\frac{(1+r_c)^{N}-1}{r_c}\right]}$$

FV annuity due

$R = \$100$

$n = 12 \qquad N = 12(5) = 60$

$t = 5$

$r = 0.08 \qquad r_c = \frac{0.08}{12} = 0.00\bar{6}$

$$S = 100(1+0.00\bar{6})\left[\frac{1.00\bar{6}^{60}-1}{0.00\bar{6}}\right]$$

$\simeq \$7{,}396.67$

(total deposits: $100(60) = \$6000$)