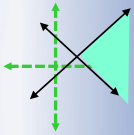
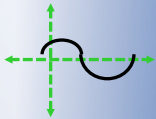


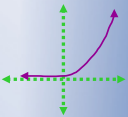
$$5x - 2y \leq 75$$



$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$



$$S = Pe^{rt}$$



$$APY = \left(1 + \frac{r}{n}\right)^n - 1$$

## Math 1090 ~ Business Algebra

### Section 4.5 Logarithmic and Exponential Equations

Objectives:

- Solve equations involving logarithmic expressions.
- Solve equations involving exponential expressions.

## Logarithmic and Exponential Equations

Strategies to solve equations:

### Logarithmic

1. Get logs on one side of the equation.
2. Condense using log properties.
3. Use the definition of a log  
to rewrite it in exponential form OR  
exponentiate both sides to undo the log.
4. Continue solving.
5. Check all answers.

domain:  
 $2x > 0$   
 $x > 0$

Ex 1: Solve these equations.

a)  $\ln(2x-3) = \ln 11$

$$\begin{aligned} \ln(2x-3) - \ln 11 &= 0 \\ \ln\left(\frac{2x-3}{11}\right) &= 0 \\ e^0 &= \frac{2x-3}{11} \\ 1 &= \frac{2x-3}{11} \\ 11 &= 2x-3 \\ 2x &= 14 \end{aligned}$$

$x = 7 \checkmark$

domain:  
 $2x-3 > 0$   
 $2x > 3$   
 $x > \frac{3}{2}$

c)  $\log_7(2x+3) = \log_7 x - \log_7 2$

$$\log_7(2x+3) - \log_7 x + \log_7 2 = 0$$

$$\log_7\left[\frac{2x+3}{x}\right] + \log_7 2 = 0$$

$$\log_7\left[\frac{2(2x+3)}{x}\right] = 0$$

$$x \cdot 7^0 = \frac{2(2x+3)}{x} \cdot x$$

$$x = 2(2x+3)$$

$$x = 4x + 6$$

$$-3x = 6$$

$$x = -2$$

### Sample Problem

$$\log_4(2x) = 3 - \log_4 8$$

$$1) \log_4(2x) + \log_4 8 = 3$$

$$2) \log_4(2x \cdot 8) = 3$$

$$\log_4(16x) = 3$$

$$3) 4^3 = 16x \text{ OR } 4^{\log_4(16x)} = 4^3$$

$$4) x = \frac{4^3}{16} = 4 \quad 16x = 4^3$$

$$5) x = 4 > 0 \checkmark$$

b)  $2 \log_4 x = 5$

$$\log_4 x = \frac{5}{2}$$

$$4^{\frac{5}{2}} = x$$

$$x = (4^{1/2})^5 = (\sqrt{4})^5 = 2^5$$

$$x = 32 \checkmark$$

domain:  $x > 0$

domain:

$$\textcircled{1} 2x+3 > 0$$

$$2x > -3$$

$$x > -\frac{3}{2}$$

and  $\textcircled{2} x > 0$

$$\Rightarrow x > 0$$

our answer

$x = -2$  does not

satisfy the domain.

$\Rightarrow$  the final answer:

N.S.

Exponential

Sample Problem

1. Isolate the exponential.
2. Use the definition of log to rewrite as a log equation OR take the log of both sides.
3. Continue solving.

$4^{x+2} = 63$

(2a)  $\ln$ 
(2b) take log
(2c) take log (of any base)

$\log_4 63 = x+2$       $\log_4 4^{x+2} = \log_4 63$       $\ln 4^{x+2} = \ln 63$

$x = -2 + \log_4 63$       $x+2 = \log_4 63$       $(x+2)\ln 4 = \ln 63$

$x+2 = \frac{\ln 63}{\ln 4}$

Change of base formula:  
 $\log_a b = \frac{\ln b}{\ln a} = \frac{\log b}{\log a}$

Ex 2: Solve these equations.

a)  $2e^x + 3 = 13$

$2e^x = 10$

(2a)  $e^x = 5$      (2b)

$\ln 5 = x$       $\ln e^x = \ln 5$

$x = \ln 5$

b)  $5^{x+6} - 4 = 12$

$5^{x+6} = 16$

(2a)     (2b)

$\log_5 16 = x+6$       $\log_5 5^{x+6} = \log_5 16$

$x+6 = \log_5 16$

$x = -6 + \log_5 16$

or  $x = -6 + \frac{\ln 16}{\ln 5}$

$\approx -4.277$

Ex 3: Solve these equations.

a)  $\log_3(2x) - \log_3(x-3) = 1$

$$\log_3\left(\frac{2x}{x-3}\right) = 1$$

(use defn log)

$$(x-3) 3^1 = \frac{2x}{x-3} (x-3)$$

$3x - 9 = 2x$ $-9 = -x$ $x = 9 \checkmark$	<p>check domain:  <math>x &gt; 0</math> and  <math>x - 3 &gt; 0</math></p>
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b)  $3^{2x} + 3^x = 20$

$$(3^x)^2 + 3^x - 20 = 0$$

of the form: if  $u = 3^x$

$$u^2 + u - 20 = 0$$

$$(u+5)(u-4) = 0$$

$$u+5=0 \text{ or } u-4=0$$

$$u=-5 \text{ or } u=4$$

$$3^x = -5 \text{ or } 3^x = 4$$

N.S. (use log defn)

$$\log_3 4 = x$$

c)  $\log(x^2) = (\log x)^2$  domain:  $x > 0$

$$2 \log x = (\log x)^2$$

$$0 = (\log x)^2 - 2 \log x$$

$$0 = \log x [\log x - 2]$$

$$\log x = 0 \text{ or } \log x - 2 = 0$$

$$10^0 = x$$

$$x = 1$$

$$\log x = 2$$

$$10^2 = x$$

$$x = 100$$

d)  $\log(x^2-x) + \log 2 - \log x = 1$

$$\log(2(x^2-x)) - \log x = 1$$

$$\log\left(\frac{2(x^2-x)}{x}\right) = 1$$

$$10^1 = \frac{2x^2 - 2x}{x}$$

$$10 = \frac{2x^2}{x} - \frac{2x}{x}$$

$$10 = 2x - 2$$

$$12 = 2x$$

$$x = 6$$

check:

$$\log(6^2 - 6)$$

$$+ \log 2$$

$$- \log 6$$

$$= \log(30)$$

$$+ \log(2)$$

$$- \log(6)$$

$$= \log\left(\frac{30 \cdot 2}{6}\right)$$

$$= \log 10 = 1 \checkmark$$