Math 1090 ~ Business Algebra

Section 4.5 Logarithmic and Exponential Equations

Objectives:

- Solve equations involving logarithmic expressions.
- Solve equations involving exponential expressions.

Logarithmic and Exponential Equations

Strategies to solve equations:

Logarithmic

1. Get logs on one side of the equation.
2. Condense using log properties.
3. Use the definition of a log to rewrite it in exponential form OR exponentiate both sides to undo the log.
4. Continue solving.
5. Check all answers. *

domain:
$2x > 0$
$x > 0$

Sample Problem

$\log_4(2x) = 3 - \log_4 8$

1) $\log_4(2x) + \log_4 8 = 3$

2) $\log_4(2x \cdot 8) = 3$

$\log_4(16x) = 3$

3) $4^3 = 16x$ OR $\cancel{4^{\log_4}}(16x) = 4^3$

$16x = 4^3$

4) $x = \frac{4^3}{16} = 4$

5) $x = 4 > 0$ ✓

Ex 1: Solve these equations.

a) $\ln(2x-3) = \ln 11$

$\ln(2x-3) - \ln 11 = 0$

$\ln\left(\frac{2x-3}{11}\right) = 0$

$e^0 = \frac{2x-3}{11}$

$1 = \frac{2x-3}{11}$

$11 = 2x - 3$

$2x = 14$

$\boxed{x = 7}$ ✓

domain:
$2x - 3 > 0$
$2x > 3$
$x > \frac{3}{2}$

b) $2\log_4 x = 5$

$\log_4 x = \frac{5}{2}$

$4^{5/2} = x$

$x = \left(4^{1/2}\right)^5 = \left(\sqrt{4}\right)^5 = 2^5$

$\boxed{x = 32}$ ✓

domain: $x > 0$

c) $\log_7(2x+3) = \log_7 x - \log_7 2$

$\log_7(2x+3) - \log_7 x + \log_7 2 = 0$

$\log_7\left[\frac{(2x+3)}{x}\right] + \log_7 2 = 0$

$\log_7\left[\frac{2(2x+3)}{x}\right] = 0$

$x \cdot 7^0 = \frac{2(2x+3)}{x} \cdot x$

$x = 2(2x+3)$

$x = 4x + 6$

$-3x = 6$

$x = -2$

domain:
① $2x + 3 > 0$
$2x > -3$
$x > -\frac{3}{2}$
and ② $x > 0$

$\Rightarrow x > 0$

our answer $x = -2$ does not satisfy the domain.

$\Rightarrow$ the final answer: $\boxed{\text{N.S.}}$

Exponential

1. Isolate the exponential.
2. Use the definition of log to rewrite as a log equation OR take the log of both sides.
3. Continue solving.

Sample Problem

$$4^{x+2} = 63$$

defn log (2a):

$$\log_4 63 = x+2$$

(3) $x = -2 + \log_4 63$

(2b) take log:

$$\log_4 4^{x+2} = \log_4 63$$
$$x+2 = \log_4 63$$

(2c) take log (of any base):

$$\ln 4^{x+2} = \ln 63$$
$$(x+2)\ln 4 = \ln 63$$
$$x+2 = \frac{\ln 63}{\ln 4}$$

Change of base formula:

$$\boxed{\log_a b = \frac{\ln b}{\ln a} = \frac{\log b}{\log a}}$$

Ex 2: Solve these equations.

a) $2e^x + 3 = 13$

$$2e^x = 10$$
$$e^x = 5$$

(2a) $\ln 5 = x$

(2b) $\ln e^x = \ln 5$

$$\boxed{x = \ln 5}$$

b) $5^{x+6} - 4 = 12$

$$5^{x+6} = 16$$

(2a) $\log_5 16 = x+6$

(2b) $\log_5 5^{x+6} = \log_5 16$

$$x+6 = \log_5 16$$
$$x = -6 + \log_5 16$$

or $x = -6 + \dfrac{\ln 16}{\ln 5}$

$\simeq -4.277$

Ex 3: Solve these equations.

a) $\log_3(2x) - \log_3(x-3) = 1$

$\log_3\left(\frac{2x}{x-3}\right) = 1$

(use defn log)

$(x-3)\ 3^1 = \frac{2x}{\cancel{x-3}}\cancel{(x-3)}$

$3x - 9 = 2x$

$-9 = -x$

$x = 9$ ✓

check domain: $x > 0$ and $x - 3 > 0$

b) $3^{2x} + 3^x = 20$

$(3^x)^2 + (3^x) - 20 = 0$

of the form: if $u = 3^x$

$u^2 + u - 20 = 0$

$(u+5)(u-4) = 0$

$u + 5 = 0$ or $u - 4 = 0$

$u = -5$ or $u = 4$

$3^x = -5$ or $3^x = 4$

N.S. (use log defn)

$\log_3 4 = x$

c) $\log(x^2) = (\log x)^2$

domain: $x > 0$

$2\log x = (\log x)^2$

$0 = (\log x)^2 - 2\log x$

$0 = \log x[\log x - 2]$

$\log x = 0$ or $\log x - 2 = 0$

$10^0 = x$

$x = 1$

$\log x = 2$

$10^2 = x$

or $x = 100$

d) $\log(x^2 - x) + \log 2 - \log x = 1$

$\log(2(x^2 - x)) - \log x = 1$

$\log\left(\frac{2(x^2-x)}{x}\right) = 1$

$10^1 = \frac{2x^2 - 2x}{x}$

$10 = \frac{2x^2}{x} - \frac{2x}{x}$

$10 = 2x - 2$

$12 = 2x$

$x = 6$

check: $\log(6^2 - 6) + \log 2 - \log 6$

$= \log(30) + \log(2) - \log(6)$

$= \log\left(\frac{30(2)}{6}\right)$

$= \log 10 = 1$ ✓