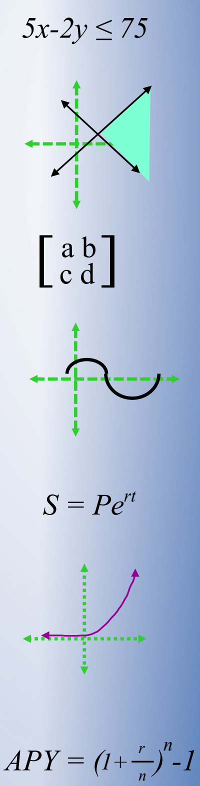

Math 1090 ~ Business Algebra

Section 4.4 Properties of Logarithms

Objectives:

- Apply the properties of logarithms to expand and contract logarithmic expressions.
- Evaluate logarithmic expressions.

Properties of Logarithms

$a>0,\ a\neq 1$

1. $\log_a a^x = x$
2. $\log_a a = 1$
3. $\log_a 1 = 0 \iff a^0 = 1$
4. $a^{\log_a x} = x$
5. $\log_a(mn) = \log_a m + \log_a n$
6. $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$
7. $\log_a m^n = n\log_a m$

WARNING!

$\log_a(m+n) \neq \log_a m + \log_a n$

i.e. logarithms NEVER distribute through addition/subtraction!!

5 claim: $\log_a(mn) = \log_a m + \log_a n$

Proof

Let $p = \log_a m$ and $q = \log_a n$

Then, equivalently $a^p = m$ and $a^q = n$.

So, $a^{p+q} = a^p a^q$ (rules of exponents)

$a^{p+q} = mn$

$\iff \log_a(mn) = p+q$

$\boxed{\log_a(mn) = \log_a m + \log_a n}$.

(5) $\log_a(mn) = \log_a m + \log_a n$

(6) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

(7) $\log_a m^n = n\log_a m$

Ex 1: Use log properties to expand.

a) $\ln\left(\frac{x^2}{x+1}\right) \overset{6}{=} \ln x^2 - \ln(x+1)$

$\overset{7}{=} 2\ln x - \ln(x+1)$

b) $\log_3(x^3\sqrt{x-2})$

$\overset{5}{=} \log_3(x^3) + \log_3\sqrt{x-2}$

$= \log_3(x^3) + \log_3(x-2)^{1/2}$

$\overset{7}{=} 3\log_3 x + \frac{1}{2}\log_3(x-2)$

c) $\log\left(\frac{y^4}{(y-2)^6}\right) \overset{6}{=} \log(y^4) - \log(y-2)^6$

$\overset{7}{=} 4\log y - 6\log(y-2)$

Ex 2: Use log properties to condense.

a) $\log_4 8 - \frac{1}{2}\log_4 5 + \log_4 3$

$\overset{7}{=} \log_4 8 - \log_4 5^{1/2} + \log_4 3$

$\overset{6}{=} \log_4\left(\frac{8}{\sqrt{5}}\right) + \log_4 3$

$\overset{5}{=} \log_4\left(\frac{8(3)}{\sqrt{5}}\right) = \log_4\left(\frac{24}{\sqrt{5}}\right)$

b) $2(\ln x - \ln(x+5)) \overset{6}{=} 2\left(\ln\left(\frac{x}{x+5}\right)\right)$

$\overset{7}{=} \ln\left(\frac{x}{x+5}\right)^2$

(5) $\log_a(mn) = \log_a m + \log_a n$

(6) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

(7) $\log_a m^n = n\log_a m$

c) $\log(2x+1) - \frac{1}{3}\log(x-1)$

$\overset{7}{=} \log(2x+1) - \log(x-1)^{1/3}$

$= \log(2x+1) - \log\sqrt[3]{x-1}$

$\overset{6}{=} \log\left(\frac{2x+1}{\sqrt[3]{x-1}}\right)$

Ex 3: Evaluate (without a calculator).

a) $\log_7 49 + \log_5 125 - \log_2 64$

$= 2 + \log_5(5^3) - \log_2(2^6)$

$= 2 + 3\underbrace{\log_5 5}_{=1} - 6\underbrace{\log_2 2}_{=1}$

$= 2 + 3 - 6 = -1$

b) $\log_4\left(\frac{1}{64}\right) + \ln(e^7) - \log_5 1$

$= \log_4(4^{-3}) + 7 - 0$

$= -3 + 7$

$= 4$

$4^{-3} = \frac{1}{4^3}$

$= \frac{1}{64}$

$\log_5 1 = \log_5(5^0)$

$= 0$

Ex 4: If $\log_b x = 1.2$, $\log_b y = 3.1$, $\log_b z = 11.1$,

evaluate $\log_b\left(\frac{x}{y}\right) - \log_b(z^2 x)$.

6

$= \log_b x - \log_b y - \log_b(z^2 x)$

5

$= \log_b x - \log_b y - [\log_b z^2 + \log_b x]$

7

$= \cancel{\log_b x} - \log_b y - 2\log_b z - \cancel{\log_b x}$

$= -\log_b y - 2\log_b z = -3.1 - 2(11.1)$

$= -3.1 - 22.2$

$= -25.3$

Ex 5: Evaluate these expressions.

a) $e^{2\ln 5}$

$\overset{7}{=} \cancel{e^{\ln}}\, 5^2$

$= 5^2 = 25$

b) $\log_4 4^a$

$\overset{7}{=} a \log_4 4$

$= a(1) = a$

c) $\cancel{\ln e}^5$

$= 5$

d) $9^{\log_9(11)+\log_9(2)}$

$= \cancel{9^{\log_9}} 11 \, \cancel{9^{\log_9}} 2$

$= 11 \cdot 2 = 22$

$\overset{5}{=} \cancel{9^{\log_9}} (11 \cdot 2)$

$= 11 \cdot 2 = 22$