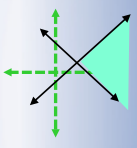
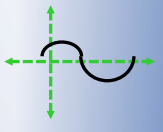
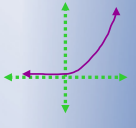
$5x-2y \leq 75$



$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$



$S = Pe^{rt}$



$APY = (1+\frac{r}{n})^{n}-1$

Math 1090 ~ Business Algebra

Section 4.1 Inverse Functions

Objectives:

- Determine if a function has an inverse.
- Determine if two functions are inverses of each other.
- Find the inverse of a one-to-one function.

Inverse Functions

An inverse function basically "undoes" what the original function did to the input, x.

Notation: $f^{-1}(x)$ is read "f inverse of x."

$$f^{-1}(f(x)) = f(f^{-1}(x)) = x$$

Ex 1: Are these functions inverses of each other?

$$f(x) = 5x - 1 \qquad g(x) = \frac{x+1}{5}$$

check: Is $f(g(x)) = x$? (or is $g(f(x)) = x$?)

$$f(g(x)) = f\left(\frac{x+1}{5}\right) = 5\left(\frac{x+1}{5}\right) - 1$$

$$= x + 1 - 1$$

$$= x \checkmark$$

$\Rightarrow f$ and g are inverse fns.

extra check: $g(f(x)) = g(5x-1)$

$$= \frac{(5x-1)+1}{5} = \frac{5x}{5} = x \checkmark$$

Does every function have an inverse? No! A function that has an inverse must pass the horizontal line test (when graphed).

One-to-one: Every input has exactly one output and every output has exactly one input.

Graphically, an inverse function is the mirror image of the original function across the line, $y = x$.

ex

Passes vert. line test ⇒ it's a fn

Passes horiz. line test ⇒ it's invertible

Ex 2: Find the inverse of $f(x) = 4(x-1)^3$.

$f^{-1}(x) = 3\sqrt{\frac{x}{4}} + 1$

① −1
② ∧3
③ ×4

Strategy to find an inverse of a function.

a) "Pants" Technique

- can only use if x shows up only once in the fn.
- undo things in opposite order in which they were done

Ex 3: Find the inverse of $f(x) = \sqrt[3]{\frac{x+1}{2x+3}}$.

$y = \sqrt[3]{\frac{x+1}{2x+3}}$

$x = \sqrt[3]{\frac{y+1}{2y+3}}$

$(2y+3)x^3 = \frac{y+1}{(2y+3)}(2y+3)$

$2x^3y + 3x^3 = y+1$

$2x^3y - y = 1 - 3x^3$

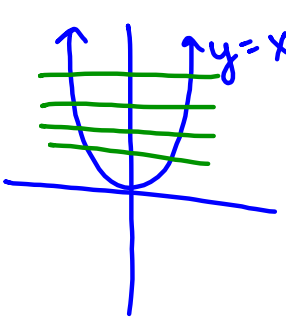
$y(2x^3-1) = 1-3x^3$

$y = \frac{1-3x^3}{2x^3-1} = f^{-1}(x)$

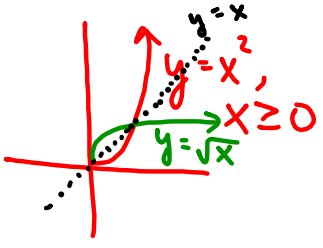
b) Standard Technique

- switch x and y
- solve for y in terms of x

Ex 4: Does $y = x^2$ have an inverse function? Can we restrict the domain so it does have an inverse function?

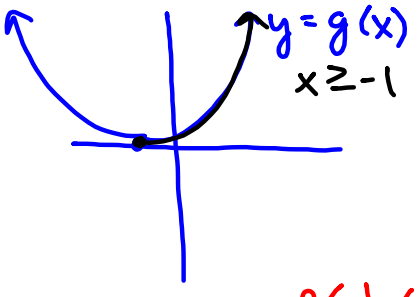


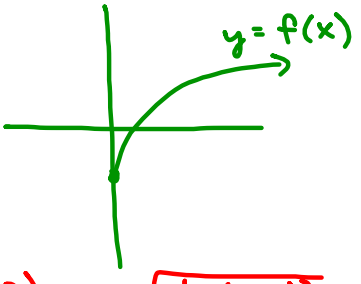
it passes vert. line test
⇒ it is a fn
but it does NOT pass horiz. line test
⇒ it's NOT invertible



this passes vert. and horiz. line tests ⇒ it's invertible

Ex 5: Are these inverses of each other? $f(x) = 2\sqrt{x} - 1$ $g(x) = \frac{1}{4}(x+1)^2$





$f(g(x)) = f\left(\frac{1}{4}(x+1)^2\right) = 2\sqrt{\frac{1}{4}(x+1)^2} - 1$

$= 2\left(\frac{1}{2}\right)(|x+1|) - 1$

$= |x+1| - 1$

$= x+1 - 1$ if $x \geq -1$

$= x$ ✓

⇒ if $x \geq -1$ for $g(x)$, then f and g are inverses.

Note
$\sqrt{(x+1)^2} = x+1$ IF $x+1$ is positive

⇒ we need $|x+1| = x+1$
(⇔ $x+1 \geq 0$
$x \geq -1$)