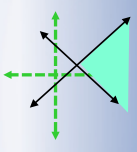
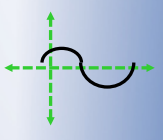
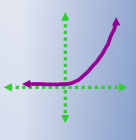
$5x-2y \leq 75$



$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$



$S = Pe^{rt}$



$APY = (1+\frac{r}{n})^{n}-1$

Math 1090 ~ Business Algebra

Section 2.5 Application Problems with Matrices

Objectives:

- Employ a variety of strategies to solve systems of equations.
- Examine an example of matrices as used in encryption.

Application Problems with Matrices

Ex 1: (Encryption)

Use $M = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 5 & -6 \\ 3 & -2 & 2 \end{bmatrix}$ to encrypt "JOYFUL" where A = 1, B = 2, etc.

A=1, B=2, C=3, D=4, ..., Y=25, Z=26

J=10, O=15, Y=25, F=6, U=21, L=12

JOYFUL becomes $\begin{bmatrix} 10 \\ 15 \\ 25 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 21 \\ 12 \end{bmatrix}$

encode the message:

$$\begin{bmatrix} 1 & -2 & 3 \\ -4 & 5 & -6 \\ 3 & -2 & 2 \end{bmatrix}\begin{bmatrix} 10 \\ 15 \\ 25 \end{bmatrix} = \begin{bmatrix} 10-30+75 \\ -40+75-150 \\ 30-30+50 \end{bmatrix} = \begin{bmatrix} 55 \\ -115 \\ 50 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 3 \\ -4 & 5 & -6 \\ 3 & -2 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 21 \\ 12 \end{bmatrix} = \begin{bmatrix} 6-42+36 \\ -24+105-72 \\ 18-42+24 \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \\ 0 \end{bmatrix}$$

⇒ encoded message: $\begin{bmatrix} 55 \\ -115 \\ 50 \end{bmatrix}\begin{bmatrix} 0 \\ 9 \\ 0 \end{bmatrix}$

the receiver of the encoded message would decode the message by multiplying by

$$M^{-1} = \frac{1}{3}\begin{bmatrix} 2 & 2 & 3 \\ 10 & 7 & 6 \\ 7 & 4 & 3 \end{bmatrix}$$

Ex 2: A grocer is going to mix three kinds of nuts to make 40 lb. of a mixture that will be priced at \$5.95/lb. The three kinds of nuts are peanuts priced at \$4.00/lb., cashews at \$6.60/lb., and pistachios at \$8.20/lb. The mixture will contain twice as much in peanuts as cashews by weight. How many pounds of each nut are in the mix?

	wt	cost/lb	total cost
peanuts	x	4.00	$4x$
cashews	y	6.60	$6.60y$
pistachios	z	8.20	$8.20z$
mix	40	5.95	$5.95(40)$

① $4x + 6.60y + 8.20z = 5.95(40)$

② $x + y + z = 40$

③ $x = 2y \iff x - 2y = 0$

note: ① is same as

$40x + 66y + 82z = 59.5(40)$

$\iff 40x + 66y + 82z = 2380$

We'll solve it using

$AX = B$

$$A = \begin{bmatrix} 40 & 66 & 82 \\ 1 & 1 & 1 \\ 1 & -2 & 0 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 2380 \\ 40 \\ 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{100}\begin{bmatrix} -2 & 164 & 16 \\ -1 & 82 & -42 \\ 3 & -146 & 26 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{100}\begin{bmatrix} -2 & 164 & 16 \\ -1 & 82 & -42 \\ 3 & -146 & 26 \end{bmatrix}\begin{bmatrix} 2380 \\ 40 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 18 \\ 9 \\ 13 \end{bmatrix} \Rightarrow$$ in mixture, we want

18 lb peanuts
9 lb cashews
13 lb pistachios

Ex 3: A company needs to borrow $150,000. For tax and related reasons, the company wants to pay 7.3% interest on this loan. There are three lenders for this money. The first charges 6%, the second charges 7% and the third charges 10%. The company is going to borrow twice as much from the first lender as from the third. How much should the company borrow from each lender?

	$ in loan	int. rate	interest ($)
①	x	0.06	$0.06x$
②	y	0.07	$0.07y$
③	z	0.10	$0.10z$
	150,000	0.073	$150000(0.073)$

(A) $x+y+z=150000$

(B) $0.06x+0.07y+0.1z = 150000(0.073)$

(C) $x=2z \iff x-2z=0$

(B) $6x+7y+10z=1{,}095{,}000$

Using substitution, $x=2z$, we get

(A) $2z+y+z=150{,}000$

(B) $6(2z)+7y+10z=1{,}095{,}000$

$\hookrightarrow$

(A) $y+3z=150{,}000$

(B) $7y+22z=1{,}095{,}000$

$$\begin{matrix} & -7 & -21 & -1050000 \\ (-7) & 1 & 3 & 150{,}000 \\ & 7 & 22 & 1{,}095{,}000 \end{matrix}$$

$$\begin{matrix} & & & \\ & 1 & 3 & 150{,}000 \\ (-3) & 0 & 1 & 45{,}000 \\ & 0 & -3 & -135{,}000 \end{matrix}$$

$$\left[\begin{array}{cc|c} 1 & 0 & 15{,}000 \\ 0 & 1 & 45{,}000 \end{array}\right]$$

$y = \$15{,}000$ (②)

$z = \$45{,}000$ (③)

$x = \$90{,}000$ (①)