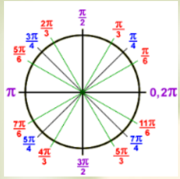
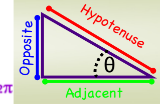
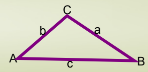
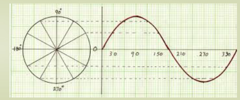








Math 1060 ~ Trigonometry

$\sin^2 u + \cos^2 u = 1$

$\sin 2u = 2\sin u \cos u$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

4 The Six Trigonometric Functions

Learning Objectives

In this section you will:

- **Determine the values of the six trigonometric functions from the coordinates of a point on the Unit Circle.**
- **Learn and apply the reciprocal and quotient identities.**
- **Learn and apply the Generalized Reference Angle Theorem.**
- **Find angles that satisfy trigonometric function equations.**

The Trigonometric Functions

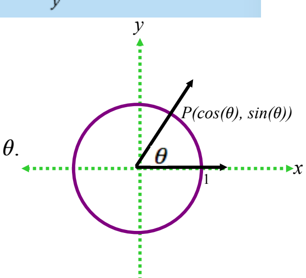
In addition to the sine and cosine functions, there are four more.

Trigonometric Functions: Suppose θ is an angle plotted in standard position and $P(x, y)$ is the point on the terminal side of θ which lies on the Unit Circle. The circular functions are defined as follows.

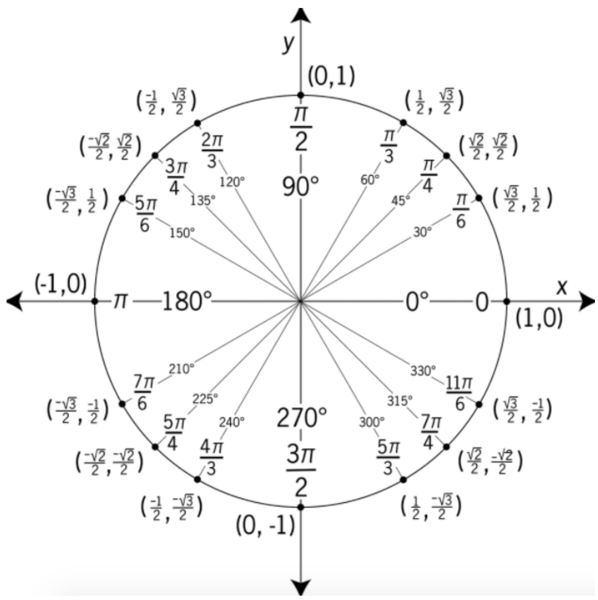
- The **sine** of θ, denoted $\sin(\theta)$, is defined by $\sin(\theta) = y$.
- The **cosine** of θ, denoted $\cos(\theta)$, is defined by $\cos(\theta) = x$.
- The **tangent** of θ, denoted $\tan(\theta)$, is defined by $\tan(\theta) = \frac{y}{x}$, provided $x \neq 0$.
- The **cosecant** of θ, denoted $\csc(\theta)$, is defined by $\csc(\theta) = \frac{1}{y}$, provided $y \neq 0$.
- The **secant** of θ, denoted $\sec(\theta)$, is defined by $\sec(\theta) = \frac{1}{x}$, provided $x \neq 0$.
- The **cotangent** of θ, denoted $\cot(\theta)$, is defined by $\cot(\theta) = \frac{x}{y}$, provided $y \neq 0$.

Ex 1: Assume θ is $\frac{\pi}{3}$ in this picture.

Find the six trigonometric functions of θ.



Ex 2: Determine the tangent values for the first quadrant and each of the quadrant angles on this Unit Circle.



Reciprocal and Quotient Identities

Reciprocal and Quotient Identities:

- $\tan(\theta)=\frac{y}{x}=\frac{\sin(\theta)}{\cos(\theta)}$, provided $\cos(\theta)\neq 0$; if $\cos(\theta)=0$ then $\tan(\theta)$ is undefined.
- $\cot(\theta)=\frac{x}{y}=\frac{\cos(\theta)}{\sin(\theta)}$, provided $\sin(\theta)\neq 0$; if $\sin(\theta)=0$ then $\cot(\theta)$ is undefined.
- $\sec(\theta)=\frac{1}{x}=\frac{1}{\cos(\theta)}$, provided $\cos(\theta)\neq 0$; if $\cos(\theta)=0$ then $\sec(\theta)$ is undefined.
- $\csc(\theta)=\frac{1}{y}=\frac{1}{\sin(\theta)}$, provided $\sin(\theta)\neq 0$; if $\sin(\theta)=0$ then $\csc(\theta)$ is undefined.

Ex 3: Find the indicated value, if it exists.

a) *sec* 30º　　　　b) $\csc \frac{11\pi}{6}$　　　　c) *cot* (2)

d) $\tan\theta$, where θ is any angle coterminal with 270 º.

e) $\cos\theta$, where $\csc\theta = -2$ and $\frac{3\pi}{2} < \theta < 2\pi$.

f) $\sin\theta$, where $\tan\theta = \sqrt{3}$ and θ is in Q III.

Generalized Reference Angle Theorem

The values of the trigonometric functions of an angle, if they exist, are the same, up to a sign, as the corresponding trigonometric functions of the reference angle.

More specifically, if α is the reference angle for θ, then $\cos\theta = \pm \cos\alpha$, $\sin\theta = \pm \sin\alpha$. The sign, + or –, is determined by the quadrant in which the terminal side of θ lies.

Ex 4: Determine the reference angle for each of these. Then state the cosine and sine and tangent of each.

a) -225º b) $\frac{11\pi}{6}$ c) - $\frac{3\pi}{4}$

Finding Angles that Satisfy Cosine and Sine Equations

Ex 5: Find all of the angles on the unit circle which satisfy the given equation.

a) $\sin\theta = 0$ b) $\cos\theta = -\frac{1}{2}$ c) $\sin\theta = \frac{\sqrt{2}}{2}$

Finding Angles that Satisfy Other Trigonometric Equations

Ex 6: Find all of the angles on the unit circle which satisfy the given equation.

a) $\tan\theta = -1$ b) $\sec\theta = 2$ c) $\cot\theta = 0$