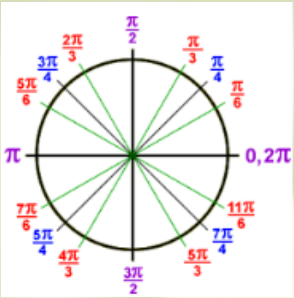
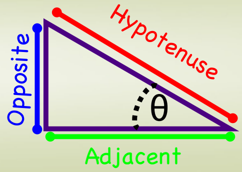
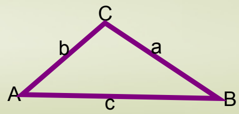
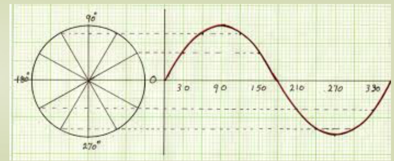








Math 1060 ~ Trigonometry

$$\sin^2 u + \cos^2 u = 1$$

$$\sin 2u = 2\sin u \cos u$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

4 The Six Trigonometric Functions

Learning Objectives

In this section you will:

- **Determine the values of the six trigonometric functions from the coordinates of a point on the Unit Circle.**
- **Learn and apply the reciprocal and quotient identities.**
- **Learn and apply the Generalized Reference Angle Theorem.**
- **Find angles that satisfy trigonometric function equations.**

The Trigonometric Functions

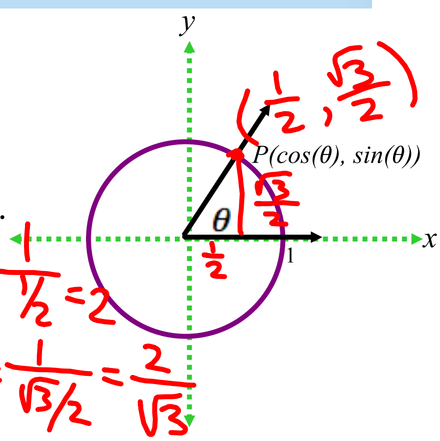
In addition to the sine and cosine functions, there are four more.

Trigonometric Functions: Suppose θ is an angle plotted in standard position and $P(x, y)$ is the point on the terminal side of θ which lies on the Unit Circle. The circular functions are defined as follows.

- The **sine** of θ, denoted $\sin(\theta)$, is defined by $\sin(\theta) = y$.
- The **cosine** of θ, denoted $\cos(\theta)$, is defined by $\cos(\theta) = x$.
- The **tangent** of θ, denoted $\tan(\theta)$, is defined by $\tan(\theta) = \frac{y}{x}$, provided $x \neq 0$.
- The **cosecant** of θ, denoted $\csc(\theta)$, is defined by $\csc(\theta) = \frac{1}{y}$, provided $y \neq 0$.
- The **secant** of θ, denoted $\sec(\theta)$, is defined by $\sec(\theta) = \frac{1}{x}$, provided $x \neq 0$.
- The **cotangent** of θ, denoted $\cot(\theta)$, is defined by $\cot(\theta) = \frac{x}{y}$, provided $y \neq 0$.

Ex 1: Assume θ is $\frac{\pi}{3}$ in this picture.

Find the six trigonometric functions of θ.



$\cos\theta = \frac{1}{2}$ $\qquad$ $\sec\theta = \frac{1}{1/2} = 2$

$\sin\theta = \frac{\sqrt{3}}{2}$ $\qquad$ $\csc\theta = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$

$\tan\theta = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$ $\qquad$ $\cot\theta = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

Ex 2: Determine the tangent values for the first quadrant and each of the quadrant angles on this Unit Circle.

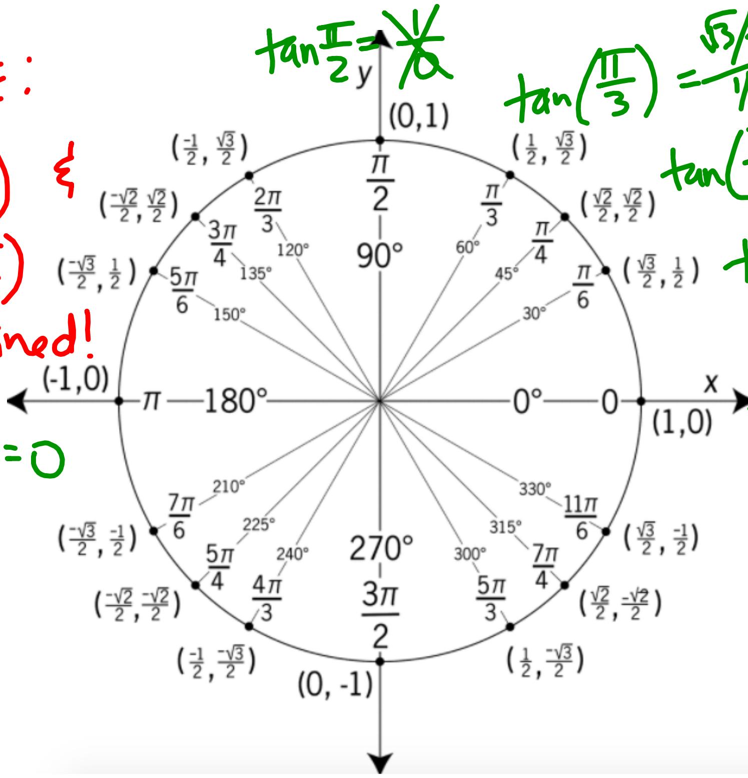
NOTE: $\tan\left(\frac{\pi}{2}\right)$ & $\tan\left(\frac{3\pi}{2}\right)$ is undefined!

$\tan\frac{\pi}{2} = \cancel{1/0}$

$\tan\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

$\tan\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$

$\tan\left(\frac{\pi}{6}\right) = \frac{1/2}{\sqrt{3}/2} = 1/\sqrt{3}$



$\tan\pi = \frac{0}{-1} = 0$

$\tan 0 = \frac{0}{1} = 0$

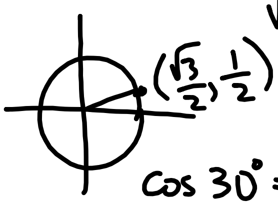
Reciprocal and Quotient Identities

Reciprocal and Quotient Identities:

- $\tan(\theta) = \frac{y}{x} = \frac{\sin(\theta)}{\cos(\theta)}$, provided $\cos(\theta) \neq 0$; if $\cos(\theta) = 0$ then $\tan(\theta)$ is undefined.
- $\cot(\theta) = \frac{x}{y} = \frac{\cos(\theta)}{\sin(\theta)}$, provided $\sin(\theta) \neq 0$; if $\sin(\theta) = 0$ then $\cot(\theta)$ is undefined.
- $\sec(\theta) = \frac{1}{x} = \frac{1}{\cos(\theta)}$, provided $\cos(\theta) \neq 0$; if $\cos(\theta) = 0$ then $\sec(\theta)$ is undefined.
- $\csc(\theta) = \frac{1}{y} = \frac{1}{\sin(\theta)}$, provided $\sin(\theta) \neq 0$; if $\sin(\theta) = 0$ then $\csc(\theta)$ is undefined.

Ex 3: Find the indicated value, if it exists.

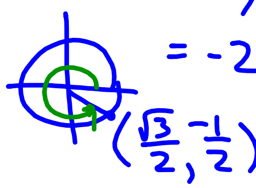
a) *sec* 30º $= \frac{2}{\sqrt{3}}$



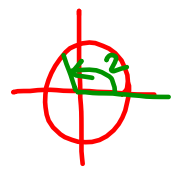
$\cos 30^\circ = \frac{\sqrt{3}}{2}$

b) *csc* $\frac{11\pi}{6} = \frac{1}{-1/2}$

$= -2$

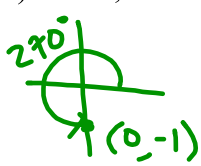


c) *cot* (2) $= \frac{\cos 2}{\sin 2}$



$\simeq -0.458$

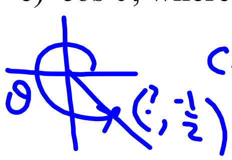
d) *tan θ*, where *θ* is any angle coterminal with 270 º.



$\tan(270^\circ)$ is undefined

$\Rightarrow \tan(\theta)$ is undefined (where θ is coterminal w/ 270°)

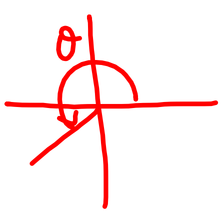
e) *cos θ*, where *csc θ* = -2 and $\frac{3\pi}{2} < \theta < 2\pi$.



$\csc\theta = -2 \Rightarrow \sin\theta = \frac{-1}{2}$

$\cos\theta = \frac{\sqrt{3}}{2}$

f) *sin θ*, where *tan θ* = $\sqrt{3}$ and *θ* is in Q III.



$\tan\theta = \sqrt{3} = \frac{-\sqrt{3}/2}{-1/2}$

$\Rightarrow \cos\theta = \frac{-1}{2}$

$\boxed{\sin\theta = \frac{-\sqrt{3}}{2}}$

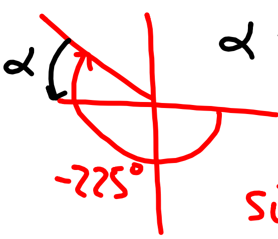
Generalized Reference Angle Theorem

The values of the trigonometric functions of an angle, if they exist, are the same, up to a sign, as the corresponding trigonometric functions of the reference angle.

More specifically, if α is the reference angle for θ, then $\cos\theta = \pm\cos\alpha$, $\sin\theta = \pm\sin\alpha$. The sign, + or –, is determined by the quadrant in which the terminal side of θ lies.

Ex 4: Determine the reference angle for each of these. Then state the cosine and sine and tangent of each.

a) -225°



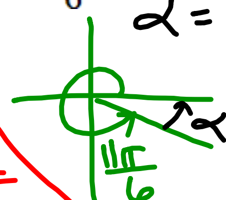
$\alpha = 45°$

$\sin(-225°) = \frac{\sqrt{2}}{2}$

$\cos(-225°) = -\frac{\sqrt{2}}{2}$

$\tan(-225°) = \frac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$

b) $\frac{11\pi}{6}$



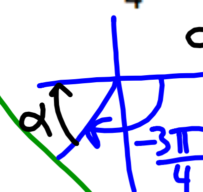
$\alpha = \frac{\pi}{6}$

$\cos\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2}$

$\tan\left(\frac{11\pi}{6}\right) = \frac{-1}{\sqrt{3}}$

c) $-\frac{3\pi}{4}$



$\alpha = \frac{\pi}{4}$

$\cos\left(\frac{-3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\sin\left(\frac{-3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

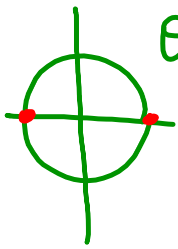
$\tan\left(\frac{-3\pi}{4}\right) = 1$

Finding Angles that Satisfy Cosine and Sine Equations

Ex 5: Find all of the angles on the unit circle which satisfy the given equation.

θ in $[0, 2\pi)$

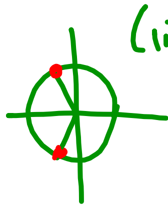
a) $\sin\theta = 0$



$\theta = 0, \pi$

b) $\cos\theta = -\frac{1}{2}$

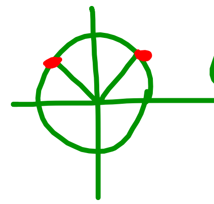
(in Q2 or Q3)



$\theta = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

c) $\sin\theta = \frac{\sqrt{2}}{2}$

(in Q1 or Q2)

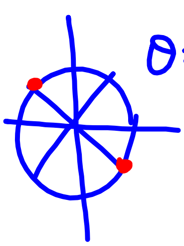


$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

Finding Angles that Satisfy Other Trigonometric Equations

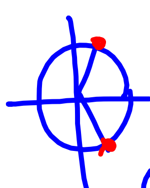
Ex 6: Find all of the angles on the unit circle which satisfy the given equation.

a) $\tan\theta = -1$ (in Q2 or Q4)



$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

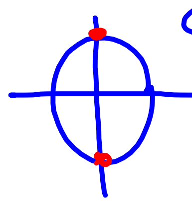
b) $\sec\theta = 2$



$\cos\theta = \frac{1}{2}$ (in Q1 or Q4)

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

c) $\cot\theta = 0$



$\Leftrightarrow \frac{\cos\theta}{\sin\theta} = \frac{0}{\pm 1}$

$\Rightarrow \cos\theta = 0$

$\theta = \pi/2$ or $\frac{3\pi}{2}$