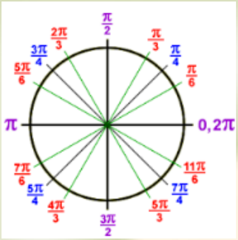
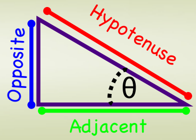
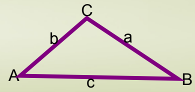
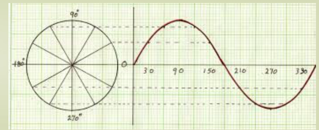


# Math 1060 ~ Trigonometry

$\sin^2 u + \cos^2 u = 1$

$\sin 2u = 2 \sin u \cos u$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

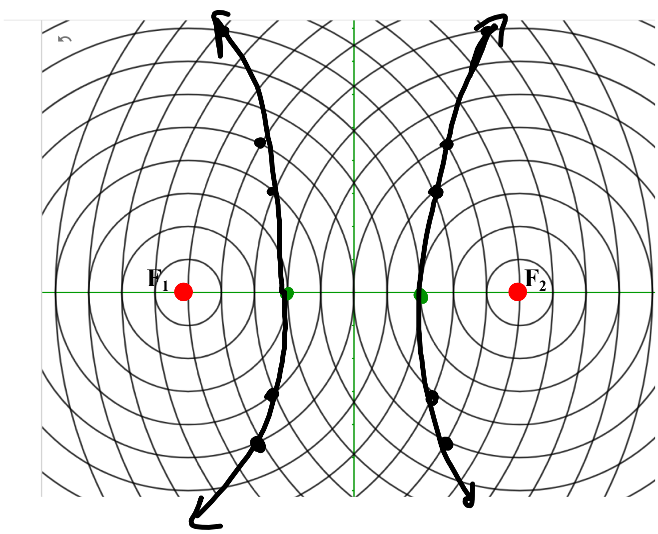
## 28 Conic Sections: Hyperbolas

### Learning Objectives

**In this section you will:**

- **Define a hyperbola in a plane.**
- **Determine whether an equation represents a hyperbola or some other conic section.**
- **Graph a hyperbola from a given equation.**
- **Determine the center, vertices, foci and eccentricity of a hyperbola.**
- **Find the equation of a hyperbola from a graph or from stated properties.**

Ex 1: Given the points $F_1(-5,0)$ and $F_2(5,0)$, plot several points such that the difference of the distances from $F_1$ and $F_2$ to each point is 4. Draw the curve connecting the points.




$7-3=4$

$8-4=4$

$9-5=4$

$12-8=4$

$e=\frac{c}{a}$

$=\boxed{\frac{5}{2}}$

Hyperbolas

conic sections

hyperbola

General form: $Ax^2 + By^2 + Cx + Dy + E = 0$, where $A$ and $B$ have opposite signs.

Given: two points (foci) and a distance ($c$).

Definition: A hyperbola is the set of all points in a plane such that for each point on the hyperbola, the difference of its distances from two fixed points is constant.

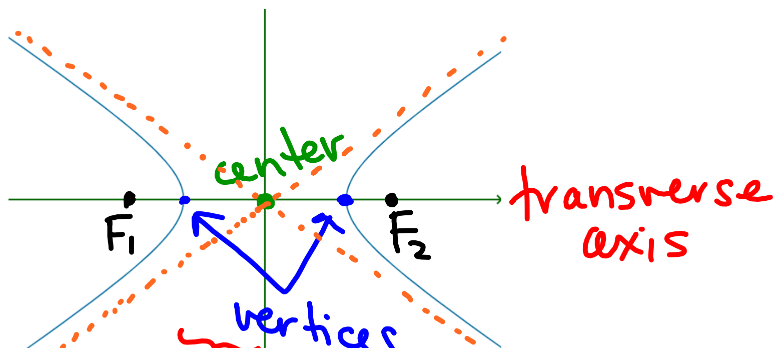
Vocabulary

Transverse axis

Asymptotes

Center

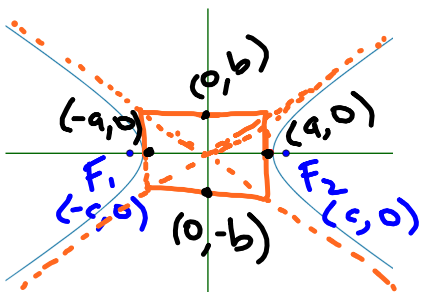
Foci




$a$ = distance from center to vertex

$c$ = distance from center to focus

Standard Form of an Equation of a Hyperbola with Center at (0,0)

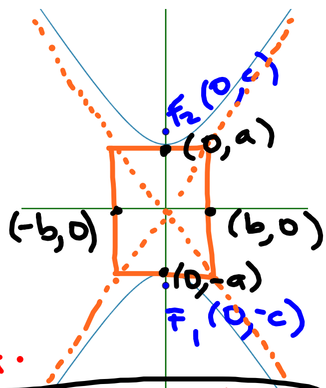



horiz.

$$\boxed{\frac{x^2}{a^2}-\frac{y^2}{b^2}=1}$$

foci $(\pm c, 0)$

vertices: $(\pm a, 0)$




vert.

$$\boxed{\frac{y^2}{a^2}-\frac{x^2}{b^2}=1}$$

foci: $(0, \pm c)$

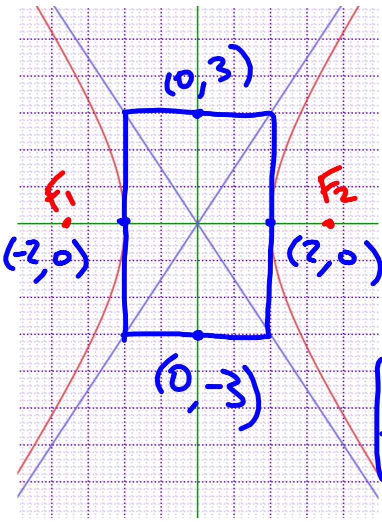
vertices: $(0, \pm a)$

The variables *a, b* and *c* have a special relationship.

$$b^2=c^2-a^2 \quad \text{OR} \quad a^2+b^2=c^2$$

Ex 2: Write the equation of these hyperbolas in standard form.

a)




$a=2$

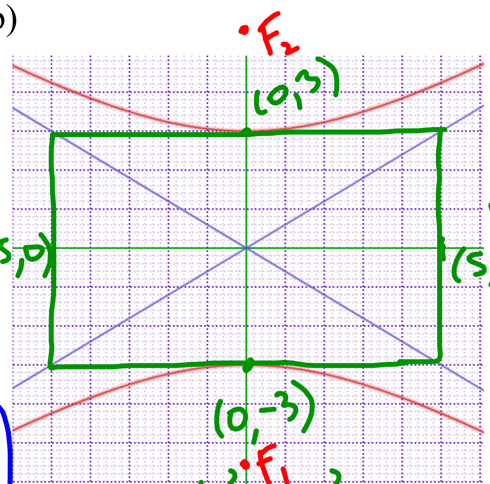
$b=3$

horiz.

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\boxed{\frac{x^2}{4}-\frac{y^2}{9}=1}$

b)




$a=3$

$b=5$

vert.

$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$

$\boxed{\frac{y^2}{9}-\frac{x^2}{25}=1}$

Ex 3: Determine the value of *c* for each hyperbola above and plot the foci.

remember $a^2+b^2=c^2$

(a) $4+9=c^2$

$c=\sqrt{13}\simeq 3.6$

$e=\frac{\sqrt{13}}{2}\simeq 1.80$

(b) $9+25=c^2$

$c^2=34$

$c=\sqrt{34}\simeq 5.8$

$e=\frac{\sqrt{34}}{3}\simeq 1.94$

Translations of a Hyperbola

Standard Hyperbola

center at (0,0)

Translated Hyperbola

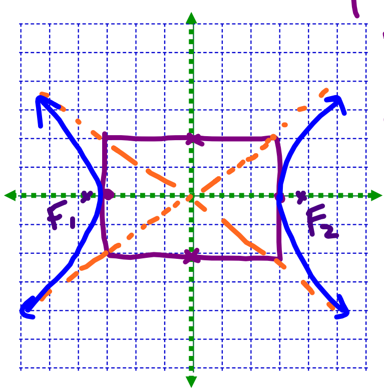
center at $(h,k)$

horiz. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\qquad \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

vert. $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ $\qquad \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

Ex 4: Sketch each of these curves and locate the foci.

a) $4x^2 - 9y^2 = 36$ $\qquad \frac{x^2}{9} - \frac{y^2}{4} = 1$




horiz.
center
(0,0)
$a = 3$
$b = 2$

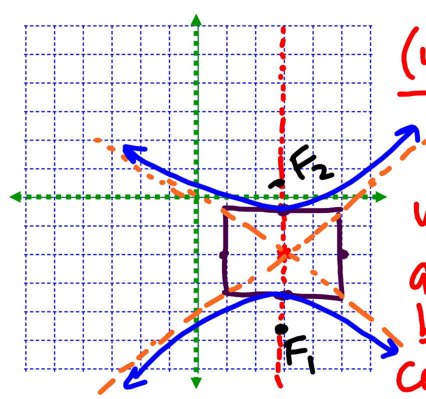
$a^2 + b^2 = c^2$

$9 + 4 = c^2 \Rightarrow c = \sqrt{13} \simeq 3.6$

$e = \frac{\sqrt{13}}{3} \simeq 1.20$

b) $16(y+2)^2 - 9(x-3)^2 = 36$

$\frac{(y+2)^2}{9/4} - \frac{(x-3)^2}{4} = 1$




vert.
$a = 3/2$
$b = 2$
center $(3,-2)$

$c^2 = \frac{9}{4} + 4 = \frac{25}{4}$

$c = \frac{5}{2}$

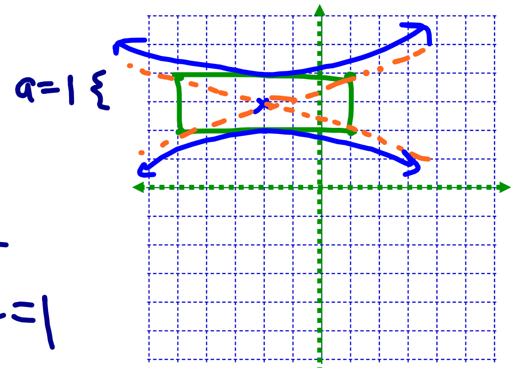
$e = \frac{5/2}{3/2} = \frac{5}{3} \simeq 1.67$

Ex 5: Write an equation and sketch each of these.

a) The hyperbola such that the center is (-2,3), one of the asymptotes passes through (1,4) and it is vertically oriented.

$a = 1$

$b = 3$




$$\frac{(y-3)^2}{1} - \frac{(x-(-2))^2}{9} = 1$$

$$\boxed{(y-3)^2 - \frac{(x+2)^2}{9} = 1}$$

$c^2 = 1+9 \Rightarrow c = \sqrt{10}$

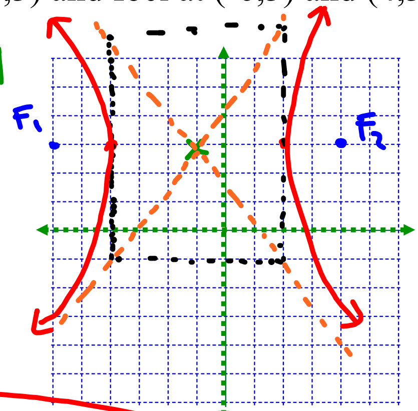
$e = \frac{\sqrt{10}}{1} \approx 3.16$

b) A hyperbola with vertices at (-4,3) and (2,3) and foci at (-6,3) and (4,3)

hyperbola is horizontal

Center (-1, 3)

$a = 3, \; c = 5$




$3^2 + b^2 = 5^2$

$9 + b^2 = 25$

$b = 4$

$$\boxed{\frac{(x+1)^2}{9} - \frac{(y-3)^2}{16} = 1}$$

$e = \frac{5}{3} \approx 1.67$

Ex 6: Write this equation in standard form, sketch it, including the foci.

$x^2 - 9y^2 - 4x - 18y - 14 = 0$

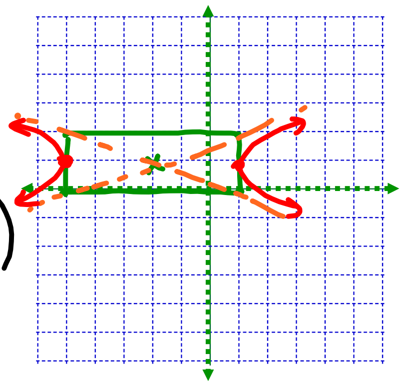
$(x^2-4x)+(-9y^2-18y)=14$

$(x^2-4x+4)-9(y^2+2y+1)$

$=14+4-9(1)$

$(x-2)^2-9(y+1)^2=9$

$$\boxed{\frac{(x-2)^2}{9}-(y+1)^2=1}$$



horiz.

$a=3$

$b=1$

center $(2,-1)$

$c^2=1+9=10 \Rightarrow c=\sqrt{10}$

$e=\frac{\sqrt{10}}{3}$

$\approx 1.05$

Eccentricity of a Hyperbola

$\boxed{e = c/a}$

for hyperbolas,

$c > a$

| | |
|---|---|
| $e=0$ | circle |
| $e=1$ | parabola |
| $0<e<1$ | ellipse |
| $e>1$ | hyperbola |

Ex 7: Identify each of these equations as one of these:

C - Circle ($x^2$ and $y^2$ have exact same coeffs.)

E - Ellipse that is not a circle (longer in which direction) ($x^2$ and $y^2$ w/ same sign coeffs.)

H - Hyperbola (facing which way) ($x^2$ and $y^2$ with opp. coeffs.)

P - Parabola (facing which way) ($x^2$ or $y^2$, but not both squared)

i) $\underbrace{9x^2 - 4y^2} - 36x + 8y - 4 = 0$

hyperbola

ii) $\underbrace{y^2} + 4x - 2y - 11 = 0$

parabola

iii) $\underbrace{16x^2 + 16y^2} + 64x - 32y - 176 = 0$

circle

iv) $\underbrace{-9x^2 + 25y^2} - 54x - 50y - 281 = 0$

hyperbola

v) $\underbrace{9x^2 + 4y^2} - 18x + 16y - 11 = 0$

ellipse

vi) $\underbrace{x^2} - 6x + 8y - 7 = 0$

parabola

vii) $\underbrace{2x^2 + 3y^2} + 12x + 24y + 60 = 0$

ellipse