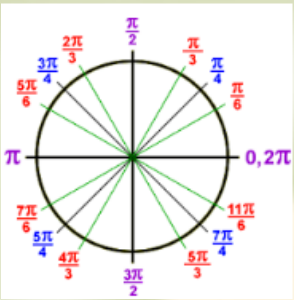
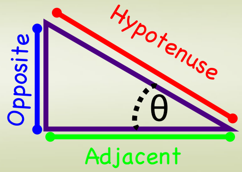
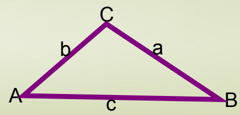
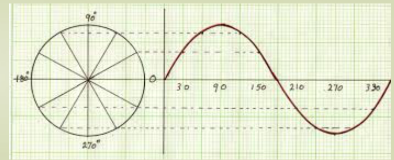








$\sin^2 u + \cos^2 u = 1$

$\sin 2u = 2 \sin u \cos u$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

Math 1060 ~ Trigonometry

26.5 Circles

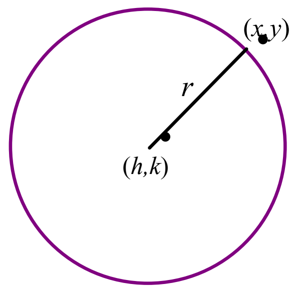
Learning Objectives

In this section you will:

In this section you will:
- Define a circle in a plane.
- Determine whether an equation represents a circle.
- Graph a circle from a given equation.
- Determine the center and radius of a circle.
- Find the equation of a circle from stated properties.

A **circle** with center (h,k) and radius $r > 0$ is the set of all points (x,y) in the plane whose distance to (h,k) is r.

a circle is the set of pts that are equidistant from a fixed pt., called the center.



$$\text{distance}^2 = (x-h)^2 + (y-k)^2$$

$$\boxed{r^2 = (x-h)^2 + (y-k)^2}$$

The **Standard Equation of a Circle** with center at (h,k) and radius $r > 0$ is $(x-h)^2 + (y-k)^2 = r^2$.

Ex 1: Write an equation of a circle with center at (2,-1) and radius 5.

$h=2,\ k=-1,\ r=5$

$$(x-2)^2 + (y+1)^2 = 5^2$$

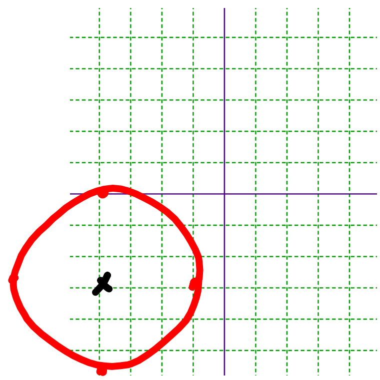
$$\boxed{(x-2)^2 + (y+1)^2 = 25}$$

Ex 2: Find the center and radius of the circle given by the equation

$(x+4)^2 + (y+3)^2 = 9$. Graph the circle.

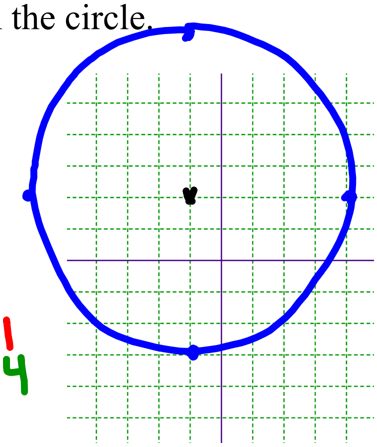
$(x-(-4))^2 + (y-(-3))^2 = 3^2$

$h = -4 \quad k = -3 \quad r = 3$



Ex 3: Put this equation in standard form and graph the circle.

$3x^2 + 3y^2 + 6x - 12y - 60 = 0$



$3(x^2 + y^2 + 2x - 4y - 20) = 0$

$(x^2 + 2x + 1) + (y^2 - 4y + 4) - 20 = 0$ $+1$ $+4$

$\left(\frac{2}{2}\right)^2 = 1 \qquad \left(\frac{-4}{2}\right)^2 = 4$

$(x^2 + 2x + 1) + (y^2 - 4y + 4) = 25$

$\boxed{(x+1)^2 + (y-2)^2 = 5^2}$ standard form of circle eqn.

center: $(-1, 2)$

radius: $r = 5$

$(x-h)^2+(y-k)^2=r^2$

Ex 4: Select the equations which might be a circle, put the equation in standard form and determine the center and radius.

a) $x^2 - y^2 + 3x - 2y - 6 = 0$

this cannot be a circle

b) $x^2 + 6x - 2y + 6 = 0$

Cannot be a circle, because it's missing y^2 term.

c) $2x^2 + 2y^2 - 4x - 10 = 0$

IS a circle

$x^2+y^2-2x-5=0$

$(x^2-2x+1)+y^2=5+1$

$(x-1)^2+y^2=6$

Center: $(1,0)$ $r=\sqrt{6}$

d) $3x + 2y - 8 = 0$

is line
not a circle

Side note:
$x^2+y^2=0$ graphs into one pt $(0,0)$

e) $x^2 + y^2 + 9 = 0$

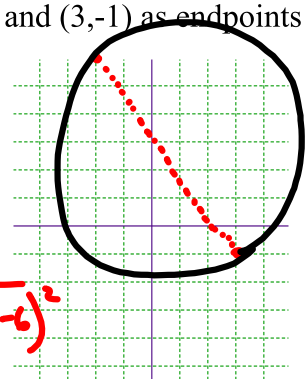
$x^2+y^2=-9$

$\Rightarrow r^2=-9$,
but that can't happen, so this is not a circle

f) $3x^2 + 2y^2 + 6x - 12y - 6 = 0$

Cannot be circle because coefficients of x^2 and y^2 are not the same

Ex 5: Write an equation of a circle with the points (-2,6) and (3,-1) as endpoints of the diameter.



① center:
midpt between $(-2,6)$ and $(3,-1)$

$\left(\frac{-2+3}{2}, \frac{6+-1}{2}\right)$

$=\left(\frac{1}{2}, \frac{5}{2}\right)$

② radius:

$r=\frac{1}{2}d$

$=\frac{1}{2}\sqrt{(3-(-2))^2+(-1-6)^2}$

$=\frac{1}{2}\sqrt{5^2+7^2}$ $=\frac{1}{2}\sqrt{74}$

(≈ 4.3)

$r^2=\left(\frac{1}{2}\right)^2\left(\sqrt{74}\right)^2$

$=\frac{74}{4}$

circle: $\boxed{\left(x-\frac{1}{2}\right)^2+\left(y-\frac{5}{2}\right)^2=\frac{74}{4}}$