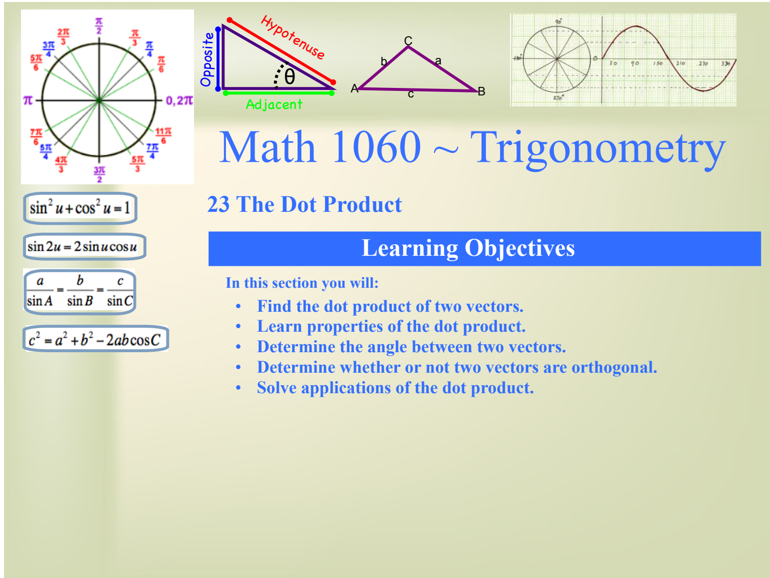


Dot Product

The dot product of two vectors is a scalar. It can be useful in finding the angle between two vectors.

If $\boldsymbol{v} = \langle v_1,v_2\rangle$ and $\boldsymbol{w} = \langle w_1,w_2\rangle$, then $\boldsymbol{v}\bullet\boldsymbol{w} = v_1w_1+v_2w_2$.

Note: $\vec{w}\cdot\vec{w} = w_1w_1+w_2w_2 = w_1^2+w_2^2 = \|\vec{w}\|^2$ i.e. $\boxed{\|\vec{w}\|^2=\vec{w}\cdot\vec{w}}$

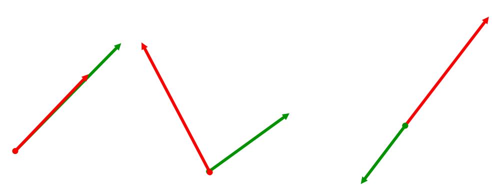
Ex 1: Find the dot product of these pairs of vectors.

a) $\boldsymbol{v} = \langle 3,4\rangle$ and $\boldsymbol{w} = \langle -2,5\rangle$.

b) $\boldsymbol{v} = \langle -3,2\rangle$ and $\boldsymbol{w} = \langle -4,-6\rangle$

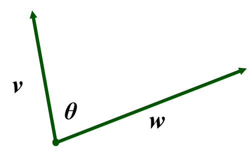
Geometric Interpretation of the Dot Product

same direction angle opposite directions



Note: there are more dot product properties listed in the book to refer to.

We will use the Law of cosines to prove that $\boldsymbol{v} \bullet \boldsymbol{w} = ||\boldsymbol{v}||\ ||\boldsymbol{w}|| \cos \boldsymbol{\theta}$, $0 < \theta < \pi$.



$\boldsymbol{v} \bullet \boldsymbol{w} = ||\boldsymbol{v}||\ ||\boldsymbol{w}|| \boldsymbol{\cos \theta}$

Ex 2: Determine the angle between these pairs of vectors.

a) $\boldsymbol{v} = \langle 3,4 \rangle$ and $\boldsymbol{w} = \langle -2,5 \rangle$.

b) $\boldsymbol{v} = \langle -3,2 \rangle$ and $\boldsymbol{w} = \langle -4,-6 \rangle$

Orthogonal vectors: If two vectors are perpendicular to each other they are said to be orthogonal. What would the cosine of the angle between two orthogonal vectors be?

Ex 3: Determine whether these pairs are vectors are orthogonal or not.

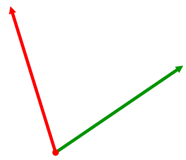
a) $\langle 3,-2 \rangle$ and $\langle 1,4 \rangle$

b) $\langle 4,-6 \rangle$ and $\langle -3,-2 \rangle$

c) $\langle 2,-1 \rangle$ and $\langle -4, 2 \rangle$

Orthogonal Projection

If $\boldsymbol{v}$ and $\boldsymbol{w}$ are nonzero vectors, then the orthogonal projection of $\boldsymbol{v}$ onto $\boldsymbol{w}$, denoted by $\text{proj}_w(\boldsymbol{v})$ is given by



$$proj_w(\vec{v}) = \left(\frac{\vec{v}\cdot\vec{w}}{\|\vec{w}\|^2}\right)\vec{w} = \left[\vec{v}\cdot\left(\frac{\vec{w}}{\|\vec{w}\|}\right)\right]\left(\frac{\vec{w}}{\|\vec{w}\|}\right)$$

$$= (\vec{v}\cdot\hat{w})\hat{w}$$

Ex 4: For $\boldsymbol{v} = \langle -6,-5 \rangle$ and $\boldsymbol{w} = \langle 10,-8 \rangle$, find $\text{proj}_w(\boldsymbol{v})$.

In physics, you will discover how this concept relates to problems about work.