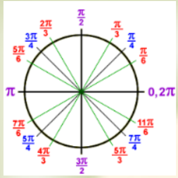
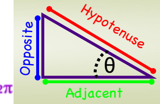
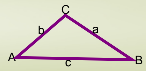
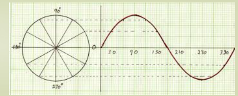








Math 1060 ~ Trigonometry

$\sin^2 u + \cos^2 u = 1$

$\sin 2u = 2 \sin u \cos u$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

18 Graphing Polar Equations

Learning Objectives

In this section you will:

- Learn techniques for graphing polar equations.
- Graph polar equations.

What do these equations represent?

$$\theta = \beta$$

$$r\cos\theta = a$$

$$r\sin\theta = b$$

What about these?

$$r = 2a\cos\theta$$

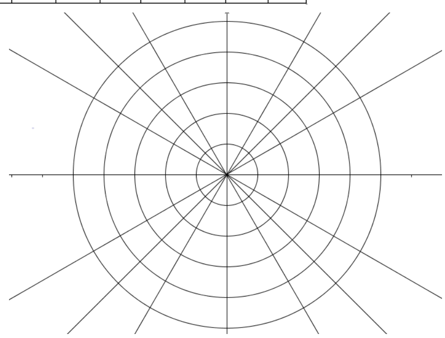
$$r = 2b\sin\theta$$

$$r = 2a\cos\theta + 2b\sin\theta$$

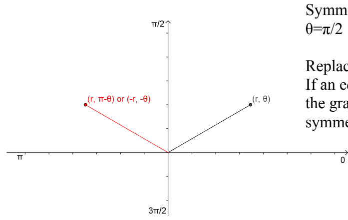
Ex 1:

$$r = 4\cos\theta$$

θ	0	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
r											



Symmetry

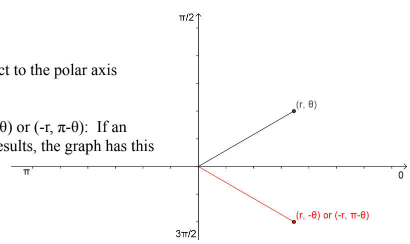


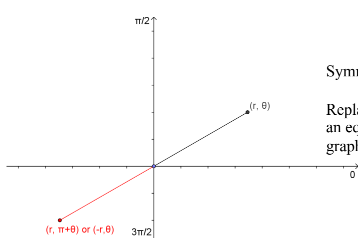
Symmetry with respect to the line θ=π/2

Replace (r,θ) with (r,π-θ) or (-r, -θ): If an equivalent equation results, the graph has this type of symmetry.

Symmetry with respect to the polar axis (θ=0):

Replace (r,θ) with (r,-θ) or (-r, π-θ): If an equivalent equation results, the graph has this type of symmetry.





Symmetry with respect to the pole

Replace (r,θ) with (-r,θ) or (r, π+θ): If an equivalent equation results, the graph has this type of symmetry.

If a polar equation passes a symmetry test, then its graph definitely exhibits that symmetry. However, if a polar equation fails a symmetry test, then its graph may or may not have that kind of symmetry.

Zeros and maximum r-values

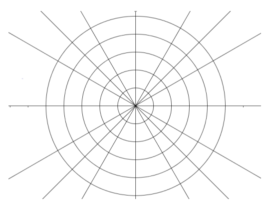
Other helpful tools in graphing polar equations are knowing the values for θ for which $|r|$ is maximum and those for which $r = 0$.

Ex 2: Graph $r = \frac{1}{2} + \cos\theta$

Symmetry:

$|r|$ maximum:

Zero of r :



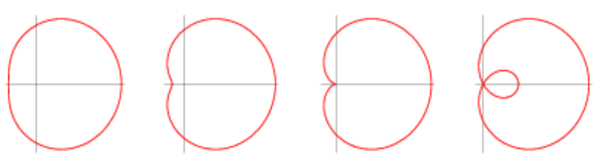
Limaçon

Limaçons

$$r = a \pm b\cos\theta$$

$$r = a \pm b\sin\theta$$

$$a > 0,\ \ b > 0$$



$\frac{a}{b} \geq 2$
Convex limaçon

$1 < \frac{a}{b} < 2$
Dimpled limaçon

$\frac{a}{b} = 1$
Cardioid -always passes through pole

$\frac{a}{b} < 1$
Limaçon with inner loop

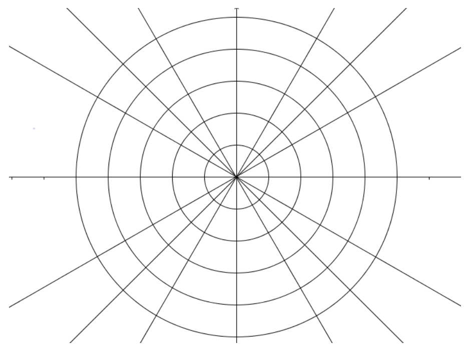
Ex 3: Graph $r = 3\sin 2\theta$

Symmetry:

r | maximum:

Zero of r :

θ	0	$\pi/8$	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
r						



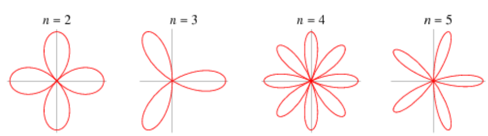
<u>Roses</u>

$$r = a\sin(n\,\theta),$$

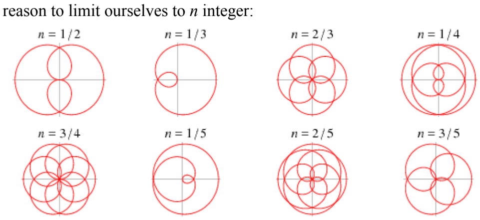
or

$$r = a\cos(n\,\theta).$$

If n is odd, the rose is n-petalled. If n is even, the rose is $2n$-petalled.



No reason to limit ourselves to n integer:



Or even rational: