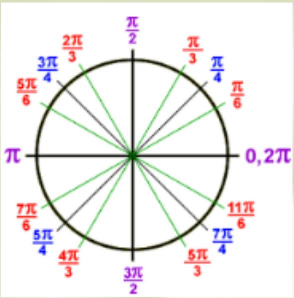
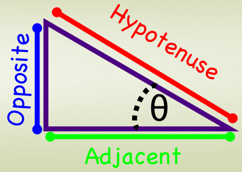
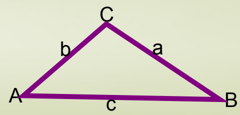
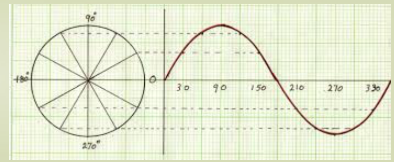








$$\sin^2 u + \cos^2 u = 1$$

$$\sin 2u = 2 \sin u \cos u$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Math 1060 ~ Trigonometry

15 The Law of Sines

Learning Objectives

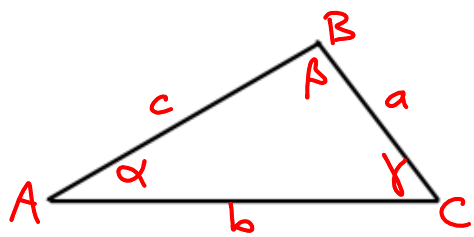
In this section you will:

- Use the Law of Sines to solve oblique triangles.
- Distinguish between ASA, AAS and SSA triangles.
- Determine the existence of, and values for, multiple solutions of oblique triangles.
- Determine when given criteria will not result in a triangle.
- Find the area of an oblique triangle using the sine function.
- Solve applied problems using the Law of Sines.

We will now apply our techniques to oblique triangles, those with no right angle.

It is important to label sides and angles of a triangle in a specific way.

Label the vertices A,B,C and the sides opposite them *a,b,c* respectively and the angles α,β,γ respectively.



The Law of Sines states that given any triangle ABC, $\boxed{\dfrac{a}{\sin\alpha}=\dfrac{b}{\sin\beta}=\dfrac{c}{\sin\gamma}}$

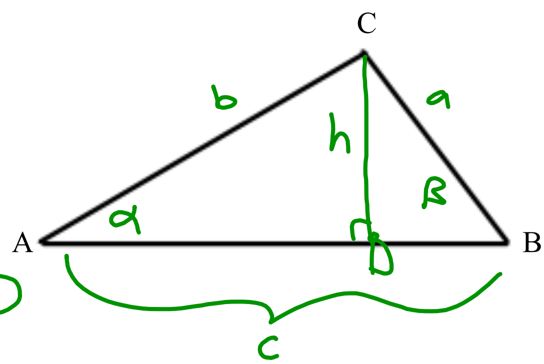
OR

It may also be stated this way: $\boxed{\dfrac{\sin\alpha}{a}=\dfrac{\sin\beta}{b}=\dfrac{\sin\gamma}{c}}$

We will prove it here.

Given: ΔABC

Prove: $\dfrac{a}{\sin\alpha}=\dfrac{b}{\sin\beta}$



Draw altitude $\overline{CD} \perp \overline{AB}$

Let CD = *h*

In Δ ACD, sin α $= \dfrac{h}{b}$ ①

In Δ BCD, sin β $= \dfrac{h}{a}$ ②

Solve each for h and set them equal to each other.

① $h = b\sin\alpha$

② $h = a\sin\beta$

$$\frac{b\cancel{\sin\alpha}}{\cancel{\sin\alpha}\sin\beta} = \frac{a\cancel{\sin\beta}}{\sin\alpha\cancel{\sin\beta}}$$

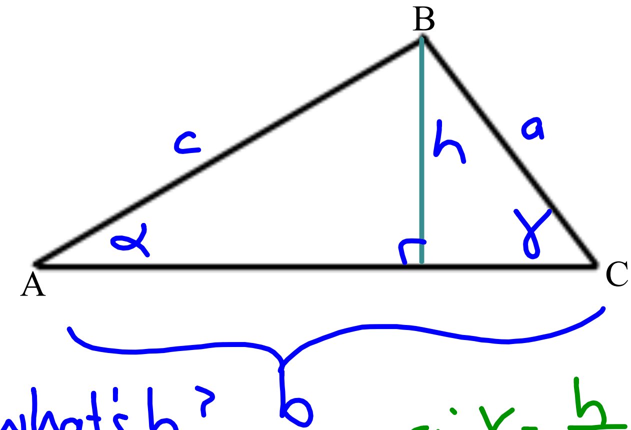
$$\frac{b}{\sin\beta} = \frac{a}{\sin\alpha} \quad ✓$$

Area of a Triangle:

There are two alternate formulas for the area of a triangle.

We will prove the first one.

$$\boxed{A = \frac{1}{2} ab \sin \gamma}$$



what we know is

$A = \frac{1}{2} bh$ but what's h?

$\sin \gamma = \frac{h}{a}$

$h = a \sin \gamma$

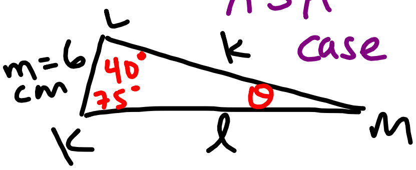
$\Rightarrow A = \frac{1}{2} b (a \sin \gamma)$

① $\boxed{A = \frac{1}{2} ab \sin \gamma}$

use this when given two leg lengths and the angle between those two sides.

② $A = \frac{1}{2} ac \sin \beta$ | ③ $A = \frac{1}{2} bc \sin \alpha$

Ex 1: Given triangle KLM, with $m = 6$ cm and the angle at L measuring 40º and the angle at K measuring 75º, solve for the remaining parts of the triangle and find the area.



ASA case

$k, \ell, \theta = ?$

$A = \frac{1}{2} k \ell \sin\theta$
$= \frac{1}{2}(6.39)(4.26)\sin 65^\circ$
$\approx \boxed{12.34 \text{ cm}^2}$ area

use law of Sines:

$$\frac{k}{\sin 75^\circ} = \frac{6 \text{ cm}}{\sin 65^\circ}$$

$$\theta = 180^\circ - 40^\circ - 75^\circ$$

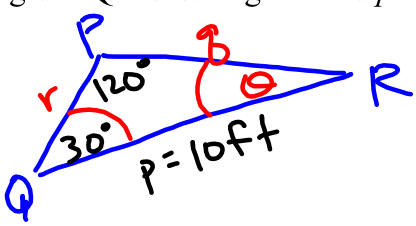
$$\boxed{\theta = 65^\circ}$$

$$k = \frac{6\sin 75^\circ}{\sin 65^\circ} \text{ cm} \approx \boxed{6.39 \text{ cm}}$$

$$\frac{\ell}{\sin 40^\circ} = \frac{6}{\sin 65^\circ} \Rightarrow \ell = \frac{6\sin 40^\circ}{\sin 65^\circ} \text{ cm} \approx \boxed{4.26 \text{ cm}}$$

Ex 2: Given triangle PQR, with the angle at P measuring 120º, the angle at Q measuring 30º and $p = 10$ ft, solve for the remaining parts.

AAS case



$r, q, \theta = ?$

use Law of Sines:

$$\frac{r}{\sin 30^\circ} = \frac{10 \text{ ft}}{\sin 120^\circ}$$

$$r = \frac{10\sin 30^\circ}{\sin 120^\circ} = \frac{10(\frac{1}{2})}{\sqrt{3}/2} = \boxed{\frac{10}{\sqrt{3}} \text{ ft}}$$

$$\theta = 180^\circ - 120^\circ - 30^\circ$$

$$\boxed{\theta = 30^\circ}$$

$\Rightarrow$ we have isosceles Δ

$\Rightarrow r = q$

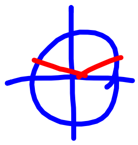
$$\boxed{r = q = \frac{10}{\sqrt{3}} \text{ ft} \approx 5.77 \text{ ft}}$$

Ex 3: Think back to your congruence postulates in Geometry, ASA, AAS, SAS, SSS and identify each problem above with its postulate.

ASA : angle, side, angle
AAS : angle, angle, side
SAS : side, angle, side
SSS : side, side, side

Let's address the dreaded SSA postulate.

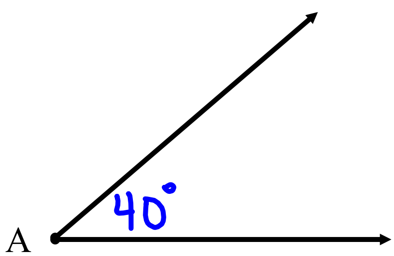
Ex 4: If $\sin(\alpha) = 0.5$ in triangle ABC, what is the measure of the angle at vertex A?

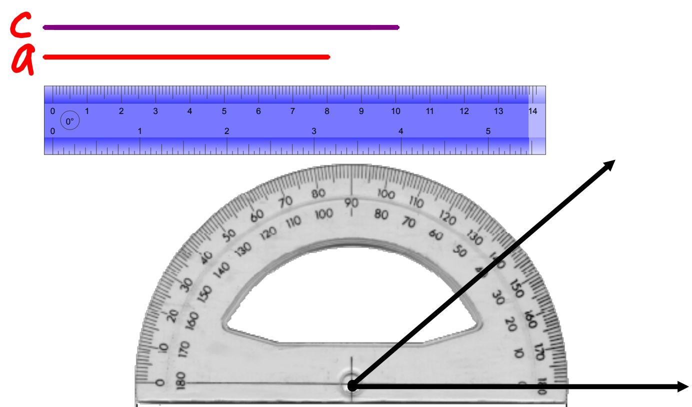


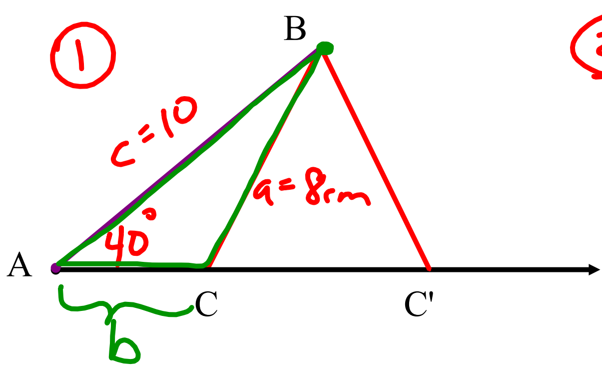
$\alpha = 30°$ or $150°$

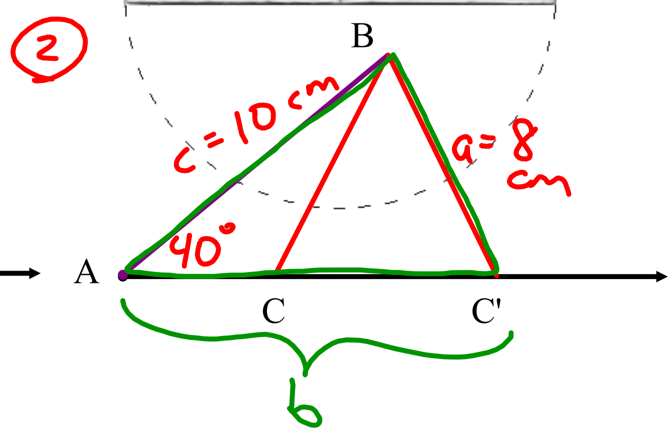
Ambiguous Case: Here is an example that leads to two different triangles in the case of SSA.

Given ΔABC with $\alpha = 40º$, $c = 10$ cm, and $a = 8$ cm, solve for the other parts.









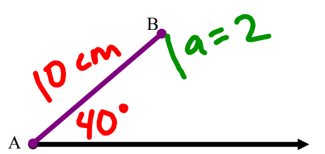
More Ambiguity

Ex 5: In the previous example, consider each of these.

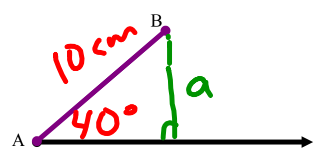
a) What if a = 2 cm?

b) Is there a value for a which produces exactly one triangle?

c) What if a = 10 cm?
$= c$



no triangle possible

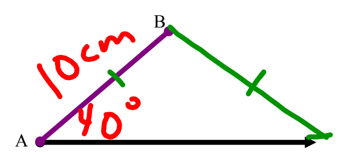


$a = ?$

$\sin 40^\circ = \frac{a}{10}$

$a = 10 \sin 40^\circ$

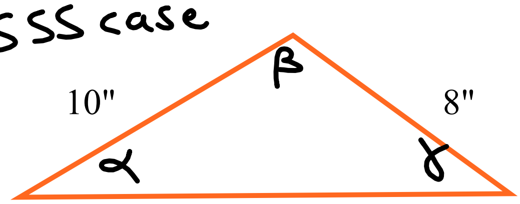
$\boxed{a \approx 6.43 \text{ cm}}$



we would get exactly one triangle

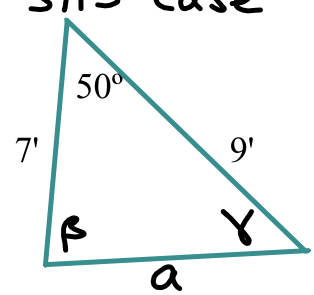
Now think about the other two postulates, SSS and SAS. Can we use the Law of Sines to solve for parts on these?

SSS case



$\frac{\sin \alpha}{8} = \frac{\sin \beta}{15}$

SAS case



$\frac{a}{\sin 50^\circ} = \frac{9}{\sin \beta}$

It becomes necessary to have another law.

** The app used in this lesson is at this link:

https://www.geogebra.org/m/CvtkyRM5