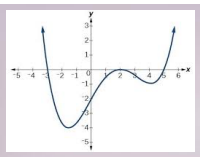
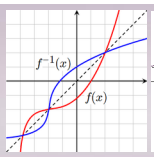
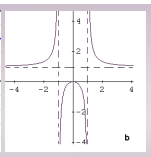
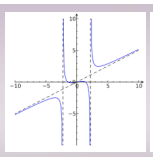
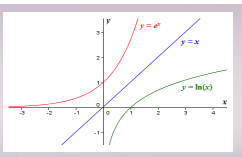



Math 1050 ~ College Algebra

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

5 Inverses of Functions

Learning Objectives

- Verify that two functions are inverses of each other.
- Determine if a function is one-to-one.
- Use the graph of a one-to-one function to graph its inverse function.
- Find the inverse of a one-to-one function.

Inverse Function

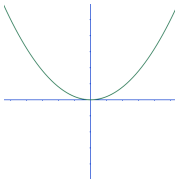
If f and g are functions such that

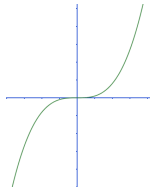
- $(f \circ g)(x) = x$ for all x in the domain of g
- $(g \circ f)(x) = x$ for all x in the domain of f

then f and g are inverses of each other.

This is written $f^{-1}(x) = g(x)$ and $g^{-1}(x) = f(x)$.

To have an inverse, a function must be one-to-one, that is for each output there must be exactly one input.





Finding an Inverse Function

Strategy

Ex 1: For $f(x)$, find the inverse function, $f^{-1}(x)$.

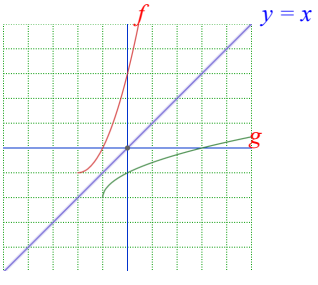
a) $f(x)=\dfrac{x^5-1}{3}$

b) $f(x)=\sqrt[3]{x+2}+1$

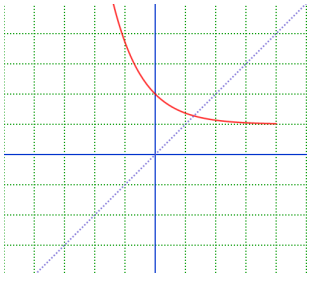
Graphical Properties of Inverse Functions

Assume f and g are inverse functions.

- The domain of f is the range of g and the domain of g is the range of f.
- $f(a) = b$ if and only if $g(b) = a$.
- (a,b) is on the graph of f if and only if (b,a) is on the graph of g.
- f and g are symmetric about the line $y = x$.



Ex 2: Sketch the inverse, f^{-1}, of f on the same axes. State the domain and range of each.

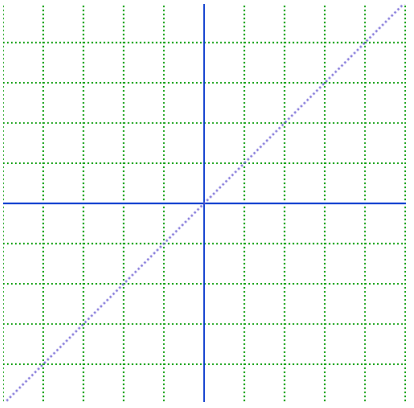


Ex 3: Show that these two functions are inverses in two ways.

$g(x)=\frac{1-x}{x},\quad 0<x\le 1$ $\qquad$ $f(x)=\frac{1}{1+x},\quad x\ge 0$

a) Algebraically

b) Graphically



Ex 4: Find the inverse of $f(x)=\frac{x-3}{x+2}$.