

# Math 1050 ~ College Algebra

## 3 Transformations of Functions

$$\begin{aligned} -3x + 4y &= 5 \\ 2x - y &= -10 \end{aligned}$$

$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^m k = \frac{m(m+1)}{2}$$

$$\sum_{k=0}^n z^k = \frac{1-z^{n+1}}{1-z}$$

### Learning Objectives

- Graph functions using vertical and horizontal shifts.
- Graph functions using reflections about the x-axis and the y-axis.
- Graph functions using vertical and horizontal scalings.
- Graph functions using a combination of transformations.

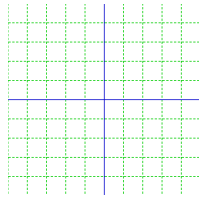
### Transformations of Functions

Types of transformations from  $y = f(x)$  to  $y = Af(Bx - C) + D$

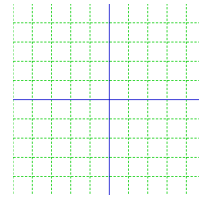
Shifts		Examples
Vertical	$h(x) = f(x) + D$	$y = x^2 + 2$
Horizontal	$g(x) = f(x-C)$	$y = (x-1)^3$
Reflect		
Vertical	$h(x) = -f(x)$	$y = -x^2$
Horizontal	$g(x) = f(-x)$	$y = \sqrt{-x}$
Stretch/shrink		
Vertical	$h(x) = A(f(x))$	$y = 5x^3$
Horizontal	$g(x) = f(Bx)$	$y = \sqrt{(\frac{1}{2})x}$

Ex 1: Graph these functions.

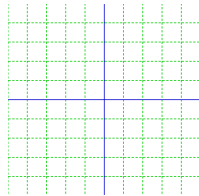
a)  $y = -\sqrt{-x}$



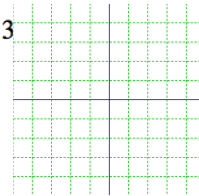
b)  $y = |x - 2| + 1$



c)  $y = -x^2 + 3$

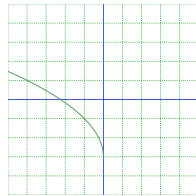


d)  $y = \frac{1}{2}(x+1)^3 - 3$

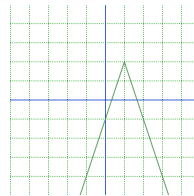


Ex 2: Write an equation for each of these graphs.

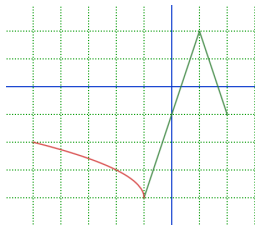
a)



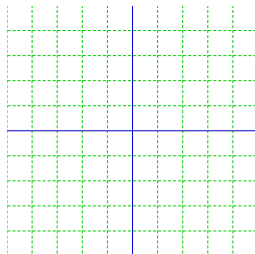
b)



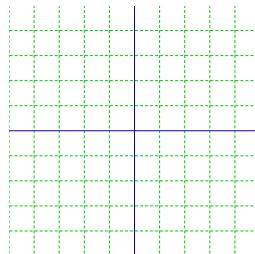
Ex 3: Given this graph for  $f(x)$ , sketch the graphs of the transformed functions.



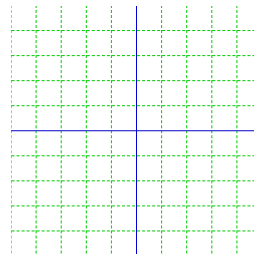
a)  $f(-x)$



b)  $f(x-1)+3$



c)  $-2f(x)$



Ex 4: Describe transformations compared to the base toolkit graph for each of these.

a)  $f(x) = 2(x+1)^3 - 9$

b)  $f(x) = -2\sqrt{x+1} + 3$

It may be helpful to use the table method to sketch a graph with several transformations. Let's look at a way to sketch this function.

$$f(x) = \left(-\frac{1}{2}x - 1\right)^3 + 3$$

Ex 5: Use the table method above to sketch this function.

$$f(x) = -3(x-2)^2 + 4$$

