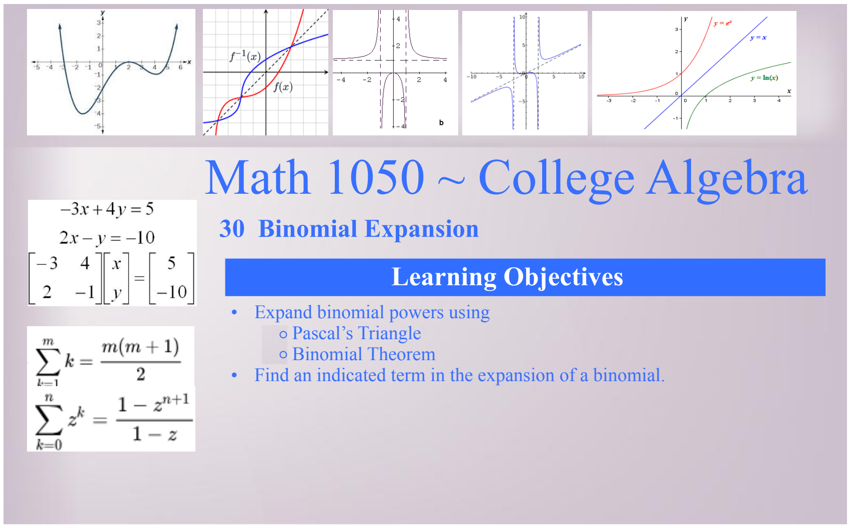

Binomial Expansion

The square and cube of a binomial are used so frequently, we may want to memorize them.

Ex 1: $(a+b)^2 =$ $\qquad\qquad$ $(a+b)^3 =$

Given these, it is possible to determine the square or cube of more complicated binomials.

Ex 2: $(x\text{-}2y)^2 =$ $\qquad\qquad$ $(3x+2y)^3 =$

Pascal's Triangle

$(a+b)^0 =$

$(a+b)^1 =$

$(a+b)^2 =$

$(a+b)^3 =$

$(a+b)^4 =$

Ex 3: Using the pattern above, expand these.

a) $(x-y)^5$

b) $(2x-3y)^4$

The Binomial Theorem

To get to this theorem, we need to introduce a new notation.

Factorial

The factorial of a non-negative integer is defined as:

$0! = 1$

$n! = 1(2)(3)...(n)$

Ex 4: Evaluate the factorials of the integers 1 through 6.

Ex 5: Evaluate

a) $\frac{5!}{3!}$ b) $(8-5)!$ c) $(8-3)!$ d) $\frac{17!}{14!}$ e) $\frac{100!}{96!4!}$

The Binomial Coefficient $\binom{n}{k}$ n, k are nonnegative integers

$n \geq k$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Ex 6: Evaluate these binomial coefficients.

a) $\binom{5}{0}$ b) $\binom{10}{6}$ c) $\binom{8}{3}$ d) $\binom{8}{5}$ e) $\binom{8}{8}$

Ex 7: Show that $(a+b)^4 = \sum_{k=0}^{4} \binom{4}{k} a^{4-k} b^k$

The Binomial Theorem $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$

Ex 8: Use this theorem to expand $(3x\text{-}2)^5$.

Ex 9: Consider $(2x\text{-}y)^8$. Find the term that contains x^3 and the term that contains y^4.