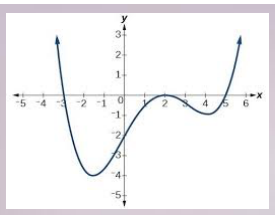
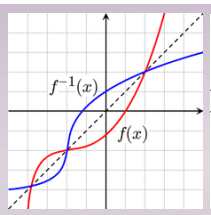
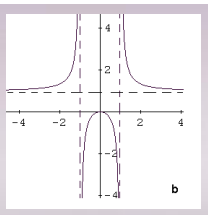
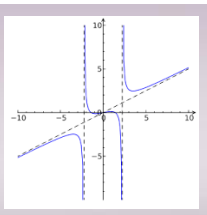
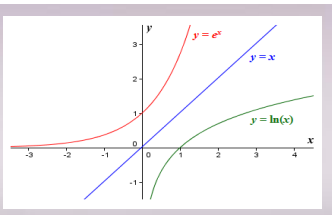





Math 1050 ~ College Algebra

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

2 Graphs of Functions

Learning Objectives

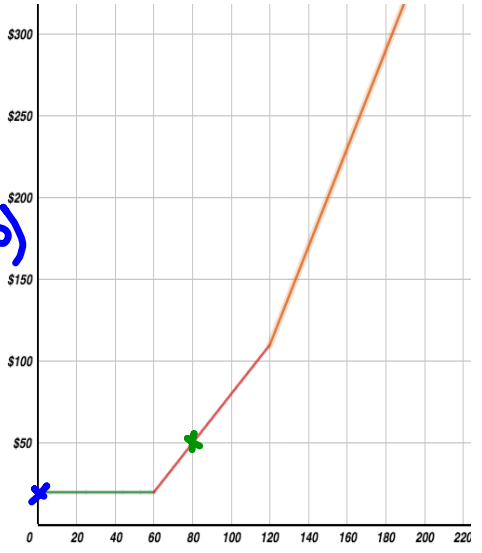
- Solve real-world applications of piecewise-defined functions.
- Identify and graph the toolkit/parent functions.
- Graph piecewise-defined functions.
- Determine whether a function is even, odd or neither.
- Determine where a function is increasing, decreasing or constant.
- Determine local maxima and minima.
- Determine absolute maximum and minimum.

Here is an example of a piece-wise function.

Ex 1: In a small community, to encourage water-wise behavior, the water company has priced it so that consumers who use more water will pay more beyond some minimum usage. After being connected to the system, the residential consumer pays a monthly flat fee of $20 until the usage exceeds 60 units. They will then pay $1.50 for each unit exceeding 60 up to 120 units, after which they will pay $3.00 per unit for those units over 120.

Here is what that function looks like, where $C(u)$ is the cost of your water in dollars and u is a unit of water (10,000 gallons).

This is a graph of the function.



$$C(u) = \begin{cases} 20 & \text{for } [0,60] \\ 1.5u - 70 & \text{for } (60,120] \\ 3u - 270 & \text{for } (120,\infty) \end{cases}$$

① $(0 \le x \le 60)$ ② ③

Verify these by using the equation and the graph.

a) C(80) = $1.5(80)-70 = \$50$ ②

cost of 80 units of water

b) C(-5) = $\$0$

(cost of -5 units of water) -5 is not in domain of C.

c) Why is C(0) = 20?

i.e. cost of 0 units of water is $20.

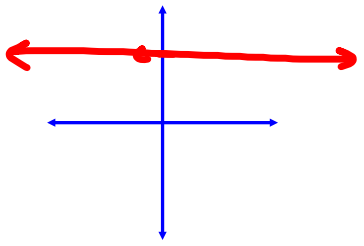
as soon as we "connect to system" we get charged $20.

The Toolkit Functions

There are several families of functions one needs to have in their toolkit.

Constant Function

$f(x) = c$ (c is a constant)



even fn

Domain: $(-\infty, \infty)$

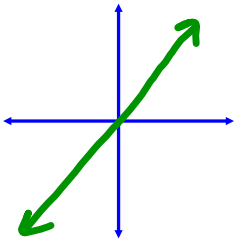
Range: $\{c\}$

x-intercept: none (if $c=0$, then ∞ many x-int.)

y-intercept: $(0, c)$

Identity Function

$f(x) = x$



odd fn

$f(-x) = -x = -f(x)$ ✓

Domain: $(-\infty, \infty)$

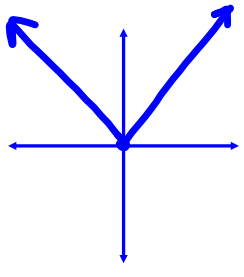
Range: $(-\infty, \infty)$

x-intercept: $(0,0)$

y-intercept: $(0,0)$

Absolute Value Function

$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$



even fn

$f(-x) = |-x| = |x| = f(x)$

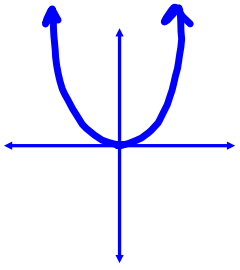
Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

x-intercept: $(0,0)$

y-intercept: $(0,0)$

Quadratic Function (parabola)



even fn

$f(x) = x^2$

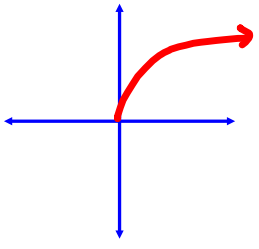
Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

x-intercept: $(0,0)$

y-intercept: $(0,0)$

Square Root Function



neither even nor odd fn

$f(x) = \sqrt{x}$

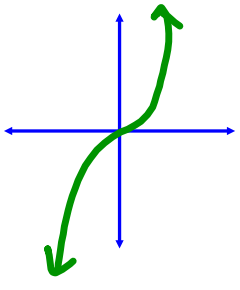
Domain: $[0, \infty)$

Range: $[0, \infty)$

x-intercept: $(0,0)$

y-intercept: $(0,0)$

Cubic Function



odd fn

$f(x) = x^3$

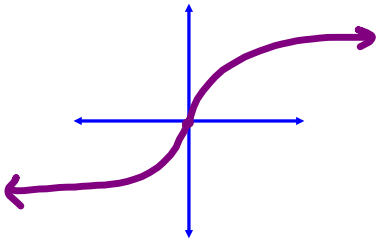
Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

x-intercept: $(0,0)$

y-intercept: $(0,0)$

Cube Root Function



odd fn

$f(x) = \sqrt[3]{x}$

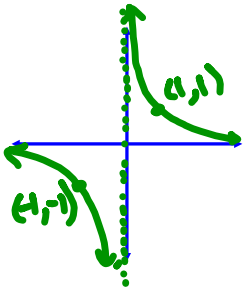
Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

x-intercept: $(0,0)$

y-intercept: $(0,0)$

Reciprocal Function



odd fn

$f(x) = \frac{1}{x}$

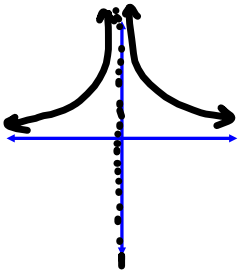
Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

x-intercept: none

y-intercept: none

Reciprocal Squared Function



even fn

$f(x) = \frac{1}{x^2}$

Domain: $(-\infty, 0) \cup (0, \infty)$

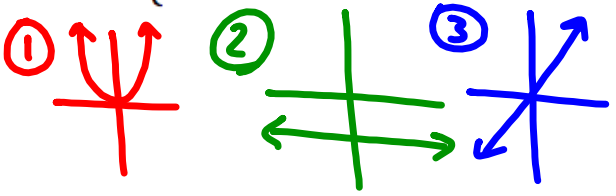
Range: $(0, \infty)$

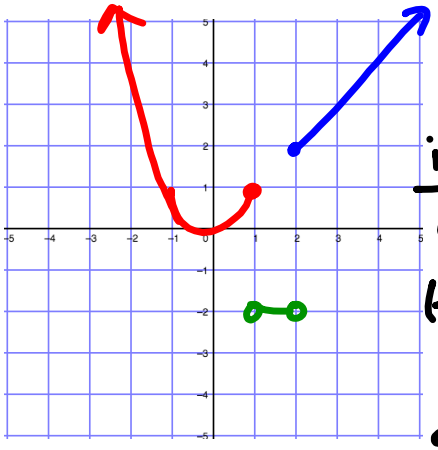
x-intercept: none

y-intercept: none

Ex 2: Graph this piece-wise function.

$$f(x)=\begin{cases} x^2 & \text{if} \quad x \le 1 \\ -2 & \text{if} \quad 1 < x < 2 \\ x & \text{if} \quad x \ge 2 \end{cases}$$





- increasing
  going up (from left to right)
- decreasing
  going down

Where is a function increasing, decreasing or constant?

Ex 3: Use the function, *f(x)* from example 2 for this exercise.

a) Use points to describe where the function is increasing, decreasing or constant.

- from left to (0,0) decreasing
- from (0,0) to (1,1) increasing
- from (1,-2) to (2,-2) constant
- from (2,2) to right increasing

b) Use domain values to describe these behaviors.

on (x-interval)

$(-\infty, 0)$ decreasing

$(0,1)$ increasing

$(1,2)$ constant

$(2, \infty)$ increasing

Determining Maximum and Minimum Function Values

Relative minimum	Relative maximum
y-value lowest locally (valley pt) ↳ min	y-value highest locally (peak) ↳ max
Absolute minimum	Absolute maximum
lowest overall y-value on the graph	highest (biggest) overall y-value on graph

Synonyms
- relative: local
- absolute: global

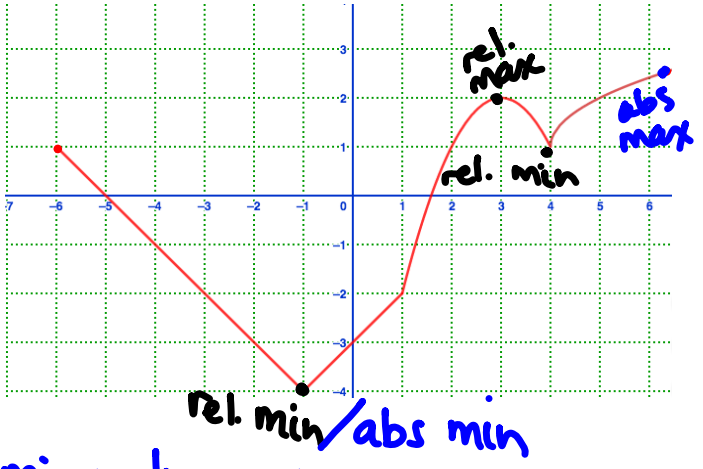
(see book for more precise defn)

Ex 4: Determine extrema values for this function.

(extrema values = min/max values)

rel. min values: -4 and 1

rel. max values: 2



abs min value: -4

abs max value: 2.5

Symmetry of Functions

Even Functions

$f(-x) = f(x)$

graphically, an even fn is symmetric about the y-axis (y-axis behaves like mirror)

Odd Functions

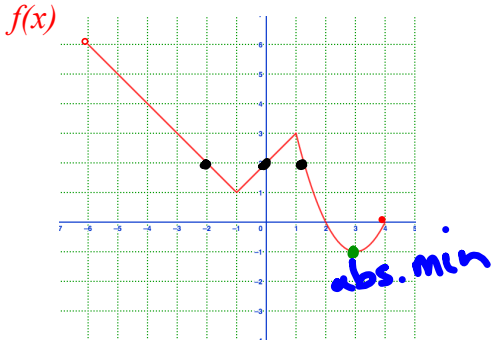
$f(-x) = -f(x)$

graphically an odd fn is symmetric about (0,0) (the origin)

Ex 5: Look at the toolkit functions and determine if any are even or odd as graphed earlier in this lesson.

(answers written on toolkit fns pages)

odd fns	even fns	neither
$y = x$	$y = c$	$y = \sqrt{x}$
$y = x^3$	$y = \lvert x \rvert$	
$y = \sqrt[3]{x}$	$y = x^2$	
$y = \frac{1}{x}$	$y = \frac{1}{x^2}$	



Ex 6: Use the function *f(x)*, represented in this graph to analyze these characteristics.

a) domain of *f* (-6, 4]

b) range of *f* [-1, 6)

c) x-intercept(s) of *f*
(2, 0) & (4, 0)

d) y-intercept of *f*
(0, 2)

e) zeros (roots) of *f*
(x-values where y = 0)
2 and 4

f) solve *f*(x) = 2 i.e. x=? when y=2
x = -2, 0, 1.2

g) *f*(3) = -1, i.e. when x=3 what is y?

h) interval(s) of increase
(-1, 1) and (3, 4)

i) maximum/minimum values
(absolute)
abs. min -1
no abs. max

j) symmetry
none