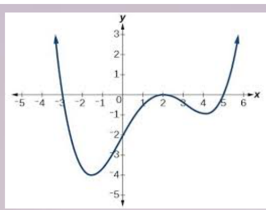

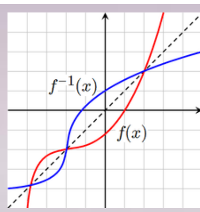

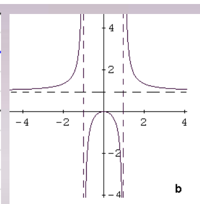

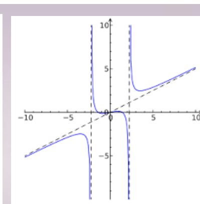

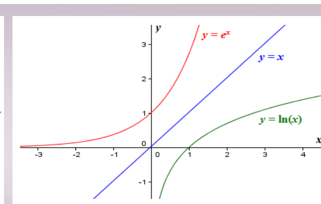

Math 1050 ~ College Algebra

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

29 Series

Learning Objectives

- Use summation notation.
- Find the sum of a finite arithmetic sequence.
- Solve applications of arithmetic series.
- Find the value of an infinite geometric series with a finite sum.
- Find the sum of a finite geometric sequence.
- Solve applications of geometric series.

Summation Notation

$\sum$ is a capital sigma

1. $\sum_{n=1}^{p} a_n = a_1 + a_2 + a_3 + \cdots + a_p$
2. $\sum_{n=j}^{p} a_n = a_j + a_{j+1} + a_{j+2} + \cdots + a_{p-1} + a_p$
3. $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots$

Ex 1: Find the following sums.

a) $\sum_{n=2}^{6}(2n-1)$

$= (2(2)-1) + (2(3)-1) + (2(4)-1)$

$n=2 \quad n=3 \quad n=4$

$+ (2(5)-1) + (2(6)-1)$

$= 3+5+7+9+11 = 35$

b) $\sum_{k=1}^{4}(-1)^k(2k)$

$= (-1)^1(2(1)) + (-1)^2(2(2))$

$+ (-1)^3(2(3)) + (-1)^4(2(4))$

$= -2+4-6+8$

$= 4$

c) $\sum_{k=0}^{5} 2^k$

$= 2^0 + 2^1 + 2^2$

$+ 2^3 + 2^4 + 2^5$

$= 1+2+4+8+16$

$+32 = 63$

Ex 2: Write the following sums using summation notation. Assume the terms in each result from an arithmetic or geometric sequence.

a) $9 - 6 + 4 - \frac{8}{3} + \frac{16}{9}$

geometric, $r = \frac{-2}{3}$

$9, -6, 4, -\frac{8}{3}, \frac{16}{9}$

$a_n = 9\left(\frac{-2}{3}\right)^{n-1}, \; n=1,\dots,5$

Sum: $\boxed{\sum_{n=1}^{5} 9\left(\frac{-2}{3}\right)^{n-1}}$

finite geom. series

b) $\frac{19}{2} + \frac{11}{2} + \frac{3}{2} - \frac{5}{2} + \cdots - \frac{29}{2}$

$n=1$

arithmetic, $d = \frac{-8}{2} = -4$

$\sum_{n=1}^{?}\left[\frac{19}{2} + (n-1)(-4)\right]$

$n=?$ when $a_n = -\frac{29}{2}$

$\frac{19}{2} + (n-1)(-4) = -\frac{29}{2}$

$-4n + 4 = -24$

$-4n = -28$

$n = 7$

$\boxed{\sum_{n=1}^{7}\left[\frac{19}{2} + (n-1)(-4)\right]}$

c) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$

geometric, $r = \frac{1}{2}$

$\sum_{n=1}^{\infty} \frac{1}{2}\left(\frac{1}{2}\right)^{n-1}$

$= \boxed{\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n}}$

Properties of Summation

1. $\sum_{n=j}^{p}(a_n \pm b_n) = \sum_{n=j}^{p} a_n \pm \sum_{n=j}^{p} b_n$

2. $\sum_{n=j}^{p} a_n = \sum_{n=j}^{h} a_n + \sum_{n=h}^{p} a_n$, for any integer $j \le h < p$

3. $\sum_{n=j}^{p} c a_n = c\sum_{n=j}^{p} a_n$ c is a constant

4. $\sum_{n=j}^{p} a_n = \sum_{n=j+h}^{p+h} a_{n-h}$ for any integer h (if $p = \infty$, replace $p+h$ with ∞)

Ex 3: Use the properties above to state these in another way.

a) $\sum_{k=1}^{8} \frac{k^2}{3}$

$\overset{3}{=} \frac{1}{3}\sum_{k=1}^{8} k^2$

b) $\sum_{k=1}^{10}\left(2k - \frac{1}{k^2}\right)$

$\overset{1}{=} \sum_{k=1}^{10}(2k) - \sum_{k=1}^{10}\frac{1}{k^2}$

c) $\sum_{j=2}^{5}(j+1) + \sum_{j=2}^{5}\frac{2}{j^2}$

$j=2,\ m=1$

$j = m+1$

$\overset{1}{=} \sum_{j=2}^{5}\left(j+1+\frac{2}{j^2}\right)$

$\overset{4}{=} \sum_{m=1}^{4}\left((m+1)+1+\frac{2}{(m+1)^2}\right)$

$= \sum_{m=1}^{4}\left(m+2+\frac{2}{(m+1)^2}\right)$

Arithmetic Series

Ex 4: Add the first hundred integers.

$$1+2+3+4+\dots+99+100=\sum_{n=1}^{100} n = 50(101) = 5050$$

$$\left.\begin{aligned} 1+100&=101\\ 2+99&=101\\ 3+98&=101\\ &\vdots\\ 50+51&=101 \end{aligned}\right\} 50 \text{ groups}$$

Qn: what is $\sum_{k=1}^{n} k$?

$$\boxed{\sum_{k=1}^{n} k = 1+2+3+\dots+n = \frac{(1+n)n}{2}}$$

$$\left.\begin{aligned} 1+n&=1+n\\ 2+(n-1)&=1+n\\ 3+(n-2)&=1+n\\ &\vdots \end{aligned}\right\} \frac{n}{2} \text{ groups}$$

Sum of a Finite Arithmetic Sequence

$$\boxed{S_n = \sum_{j=1}^{n} a_j = \frac{n}{2}(a_1 + a_n)} = \underline{\frac{n}{2}(2a_1 + (n-1)d)}, \quad n \geq 2 \qquad \text{where } a_j = a_1 + (j-1)d$$

arithmetic seq.

Ex 5: Find the n^{th} partial sum for each of these.

a) $\sum_{n=2}^{20}(2n-1)$

sum of arithmetic seq.

$m=1$ when $n=2$

$m=19$ when $n=20$

$n=m+1$

$$\sum_{n=2}^{20}(2n-1) = \sum_{m=1}^{19}(2(m+1)-1) = \sum_{m=1}^{19}(2m+1)$$

$$= \frac{19}{2}\left[(2(1)+1)+(2(19)+1)\right] = 399$$

b) $\frac{19}{2} + \frac{11}{2} + \frac{3}{2} - \frac{5}{2} + \cdots \quad , n = 10$

$d=-4$

$$= \sum_{k=1}^{10}\left[\frac{19}{2} + (k-1)(-4)\right]$$

$$= \frac{10}{2}\left[\frac{19}{2} + \frac{19}{2} + (9)(-4)\right]$$

$$= -85$$

c) The sequence $\{a_n\}$ where $a_1 = 15$, $a_{10} = 312$, $n = 50$

(assume it's arithmetic)

$a_n = 15 + (n-1)d$

$312 = 15 + 9d$

$297 = 9d \Leftrightarrow d = 33$

$a_n = 15 + (n-1)(33)$

$a_n = 33n - 18$

$$\rightarrow \sum_{n=1}^{50} a_n = \frac{50}{2}(a_1 + a_{50}) = 25(15 + (33(50) - 18))$$

$$= 41{,}175$$

Sum of a Finite Geometric Sequence

$$\boxed{S_n = \sum_{j=1}^{n} a_j = a_1 \frac{(1-r^n)}{1-r}}$$ where $a_j = a_1(r^{j-1})$

$$a_1 + \cancel{a_1r} + \cancel{a_1r^2} + \cancel{a_1r^3} + \ldots + \cancel{a_1r^{n-1}} = S_n$$
$$-(\quad \cancel{a_1r} + \cancel{a_1r^2} + \cancel{a_1r^3} + \ldots + \cancel{a_1r^{n-1}} + a_1r^n = rS_n)$$
$$a_1 - a_1r^n = S_n - rS_n$$
$$a_1(1-r^n) = S_n(1-r) \iff S_n = \frac{a_1(1-r^n)}{1-r} \quad r \neq 1$$

Sum of an Infinite Geometric Sequence

$$\boxed{S = \sum_{j=1}^{\infty} a_j = \frac{a_1}{1-r}}, \; -1 < r < 1$$ where $a_j = a_1(r^{j-1})$

Ex 6: Determine each sum.

a) $\sum_{n=0}^{\infty}\left(\frac{1}{10}\right)^n$

geometric series, $r = \frac{1}{10}$, $a_1 = 1$

$$= \frac{1}{1-\frac{1}{10}} = \frac{1}{9/10} = \frac{10}{9}$$

b) $\sum_{n=0}^{\infty} 2\left(\frac{2}{3}\right)^n$

geometric series. $r = \frac{2}{3}$, $a_1 = 2$

$$= \frac{2}{1-\frac{2}{3}} = \frac{2}{1/3} = 6$$

c) $1.3\overline{8}$ Hint: $1.3\overline{8} = 1.3 + 0.08 + 0.008 + 0.0008 + \ldots$

$$1.3\overline{8} = 1.3 + \frac{8}{10^2} + \frac{8}{10^3} + \frac{8}{10^4} + \ldots$$
$$= 1.3 + \sum_{n=2}^{\infty} \frac{8}{10^n} = 1.3 + \sum_{n=2}^{\infty} 8\left(\frac{1}{10}\right)^n$$

geom. series $r = \frac{1}{10}$

$$= 1.3 + \frac{\text{first term}}{1-\frac{1}{10}} = 1.3 + \frac{\frac{8}{100}}{9/10}$$
$$= 1.3 + \frac{8}{10\cancel{0}} \cdot \frac{\cancel{10}}{9} = \frac{13}{10} + \frac{8}{90} = \frac{13(9)+8}{90}$$
$$= \frac{125}{90} = \frac{25}{18}$$

d) $\sum_{k=0}^{5} 3^k$

finite sum of geom. seq.

$r = 3$

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right) = \sum_{k=1}^{n} a_k$$

$$\sum_{k=0}^{5} 3^k = \sum_{m=1}^{6} 3^{m-1}$$

$k=0$, $m=1$
$m=k+1$
$k=5$, $m=6$

$$= 1\left(\frac{1-3^6}{1-3}\right)$$
$$= \frac{-728}{-2} = \boxed{364}$$

Applications of Series

Ex 7: You are trying to break a bad habit. Two relatives offer to help with a financial incentive, but you must choose only one. How much is each offer? Which would you take?

a) Your Great Auntie Mare offers to give you $1.00 on the first day of February and each day thereafter, she will give you one dollar more than she did the day before.

28 days

$1+2+3+4+\dots+28=\sum_{n=1}^{28} n$

sum of arith. seq.

$=\frac{28(28+1)}{2}$

$=14(29)=\$406$

b) Your Uncle Ulysses offers to give you 1 cent on the first day of February and each day thereafter, he will give you double what he gave you the day before.

$0.01+0.02+0.04+0.08$
$+\dots+0.01(2)^{27}$

sum of geom. seq.

$=\sum_{n=1}^{28}\left(0.01\left(2^{n-1}\right)\right)$

$=0.01\left(\frac{1-2^{28}}{1-2}\right)$

$=-0.01(-268{,}435{,}455)$

$=\$2{,}684{,}354.55$