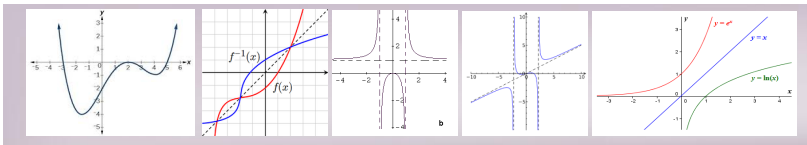

Math 1050 ~ College Algebra

$$-3x+4y=5$$
$$2x-y=-10$$
$$\begin{bmatrix}-3 & 4\\ 2 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}5\\ -10\end{bmatrix}$$

27 Partial Fractions

$$\sum_{k=1}^{m} k=\frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k=\frac{1-z^{n+1}}{1-z}$$

Learning Objectives

- Decompose a rational expression with denominator of non-repeated linear factors into a sum of partial fractions.
- Decompose a rational expression with denominator of repeated linear factors into a sum of partial fractions.
- Decompose a rational expression with denominator of non-repeated irreducible quadratic factors into a sum of partial fractions.
- Decompose a rational expression with denominator of repeated irreducible quadratic factors into a sum of partial fractions.

Partial Fraction Decomposition

Distinct Linear Factors

There are times, in future math classes, when you would like to break a rational expression into a sum of simpler fractions. We will begin with a proper fraction, where the degree of the numerator is less than the degree of the denominator. The first step is to factor the denominator and write it as a sum of n terms for an n^{th} degree denominator. $\frac{p(x)}{q(x)}=\frac{A}{a_1x+b_1}+\frac{B}{a_2x+b_2}+\frac{C}{a_3x+b_3}+\cdots$

Ex 1: Determine A and B for this proper fraction. $\frac{3x-1}{x(x-4)}=\frac{A}{x}+\frac{B}{(x-4)}$

If the fraction is improper, we must do long division first.

Ex 2: Write the partial fraction decomposition for this expression. $\frac{x^2+1}{x^2-x}$

Repeated Linear Factors

$$\frac{p(x)}{q(x)} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \cdots + \frac{N}{(ax+b)^n}$$

Ex 3: Resolve into partial fractions $\frac{2x^2+7x+4}{(x+1)^3}$.

Unique Irreducible Quadratic Factors

$$\frac{p(x)}{q(x)} = \frac{Ax+B}{a_1x^2+b_1x+c_1} + \frac{Cx+D}{a_2x^2+b_2x+c_2} + \cdots$$

Ex 4: Write the partial fraction decomposition of $\frac{-x^3+4x^2-2x+6}{x^2(x^2+2)}$.

Repeated Irreducible Quadratic Factors

$$\frac{p(x)}{q(x)} = \frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{\left(ax^2 + bx + c\right)^2} + \frac{A_3x + B_3}{\left(ax^2 + bx + c\right)^3} + \cdots + \frac{A_nx + B_n}{\left(ax^2 + bx + c\right)^n}$$

Ex 4: Write the partial fraction decomposition of $\frac{x^2 + x + 2}{(x^2 + 2)^2}$.