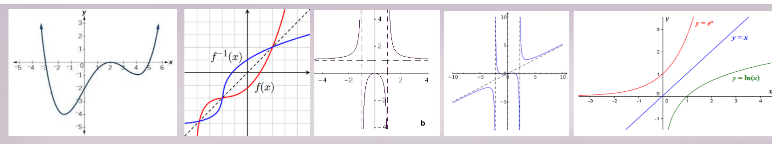

Math 1050 ~ College Algebra

26 Systems of Linear Equations: Determinants

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

Learning Objectives

- Find the determinant of a 2×2 or 3×3 matrix.
- Solve a system of linear equations using Cramer's Rule.

<u>Determinant of a Matrix</u>

Every square matrix has a number associated with it, called the determinant of A. It may be written det(A) or $|A|$.

For a 2 × 2 matrix, det(A) is given by this formula.

$$Det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

Ex 1: Find the determinant of each of these matrices.

a) $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 3 & 5 \\ 2 & 8 \end{bmatrix}$ c) $\begin{bmatrix} 3 & -1 \\ -6 & 2 \end{bmatrix}$

<u>Cramer's Rule</u>

For a set of two equations in two unknowns, Cramer's Rule says that

$$\begin{aligned} ax + by &= c \\ dx + ey &= f \end{aligned}$$ has solutions $x = \dfrac{ce - bf}{ae - bd}$, $y = \dfrac{af - cd}{ae - bd}$.

Ex 2: Use the rule above to determine the solution.

$$\begin{aligned} 2x + y &= 4 \\ 5x + 3y &= -1 \end{aligned}$$

Determinant of a 3×3 matrix is more complex.

$$A=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \Rightarrow \det(A)=|A|=a\begin{vmatrix} e & f \\ h & i \end{vmatrix}-b\begin{vmatrix} d & f \\ g & i \end{vmatrix}+c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Given the square $n \times n$ matrix A where $n > 1$, and a_{ij} represents the entry in the i^{th} row and j^{th} column:

- the minor, M_{ij} of the entry a_{ij} is the determinant of the $(n\text{-}1)\times(n\text{-}1)$ matrix left after deleting row i and column j from the matrix A.
- the cofactor, C_{ij} of entry a_{ij} is $C_{ij}=(-1)^{i+j}M_{ij}$.

Ex 3: Find all M_{ij} and C_{ij} for this matrix. $A=\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$

The determinant of an $n \times n$ matrix, where n > 1, is the sum of the entries in any row or column multiplied by each entry's respective cofactor.

Ex 4: Find the determinant of $A=\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$.

To use Cramer's Rule to solve a set of 3 equations, let $D = \det A$. D_x is found by replacing the first column of A by the constants. D_y is found by replacing the second column of A by the constants, and D_z is found by replacing the third column of A by the constants.

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}, \quad z = \frac{D_z}{D}$$

Ex 5: Use Cramer's Rule to solve.

$$\begin{aligned} x - y &= 1 \\ x - z &= -2 \\ 6x - 2y - 3z &= -4 \end{aligned}$$