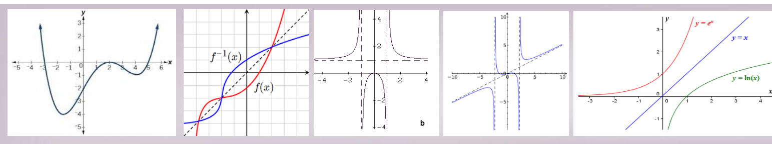

Math 1050 ~ College Algebra

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

24 Matrix Arithmetic

Learning Objectives

- Find the sum and difference of two matrices.
- Find the scalar multiple of a matrix.
- Find the product of two matrices.

<u>Definition of a Matrix</u>

A matrix is an array of numbers with m rows and n columns. The size of a matrix is described as $m \times n$. This is called the <u>dimension</u> of a matrix.

Ex 1: Determine the dimension of each matrix.

$$A = \begin{bmatrix} 1 & 0 & -5 \\ -2 & 5 & 0 \\ 1 & 4 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 7 & 3 & 6 \\ 0 & 2 & -1 & 4 \\ 3 & 5 & 1 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} -3 & 5 & 1 & 8 \end{bmatrix} \quad D = \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 0 \\ 2 & 5 & 2 \end{bmatrix} \qquad F = \begin{bmatrix} 1 & 0 \\ -2 & 4 \\ 3 & -1 \end{bmatrix} \qquad \boldsymbol{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The elements of a matrix are called <u>entries</u>. An $m \times n$ matrix has $m \cdot n$ entries. Two matrices are equal only if they are the same dimension and each corresponding element is equal.

Ex 2: If $\begin{bmatrix} x & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 4 & y \end{bmatrix}$, solve for x and y.

Operations with Matrices

Matrix Addition/Subtraction $\Longrightarrow A \pm B$

If two matrices are the same dimension, then addition or subtraction may be accomplished by adding/subtracting the corresponding entries of the two matrices.

$$A = \begin{bmatrix} 1 & 0 & -5 \\ -2 & 5 & 0 \\ 1 & 4 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 7 & 3 & 6 \\ 0 & 2 & -1 & 4 \\ 3 & 5 & 1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} -3 & 5 & 1 & 8 \end{bmatrix} \quad D = \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 0 \\ 2 & 5 & 2 \end{bmatrix} \quad G = \begin{bmatrix} 1 & 0 \\ -2 & 4 \\ 3 & -1 \end{bmatrix} \quad \boldsymbol{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \boldsymbol{O} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Ex 2: Determine which matrices from example 1 can be added or subtracted, then perform each operation on them (Exclude the Identity matrix.)

Scalar Multiplication $\Rightarrow kA$

A scalar is a real number. Multiplication of a matrix by a scalar, k is accomplished by multiplying each entry by that scalar.

Ex 3: Perform these operations.

a) $5C$ b) $-3G$ c) $2\boldsymbol{I} - 3A$

Properties of Matrix Addition

- **Commutative Property:** For all $m \times n$ matrices, $A + B = B + A$
- **Associative Property:** For all $m \times n$ matrices, $(A+B)+C = A+(B+C)$
- **Identity Property:** For all $m \times n$ matrices, $A + \mathbf{0} = \mathbf{0} + A = A$
- **Inverse Property:** Every $m \times n$ matrix A has a unique **additive inverse**, denoted $-A$, such that $A + (-A) = (-A) + A = \mathbf{0}$

Ex 4: Write an additive identity matrix and an additive inverse matrix for each of these.

a) $G = \begin{bmatrix} 1 & 0 \\ -2 & 4 \\ 3 & -1 \end{bmatrix}$ b) $C = \begin{bmatrix} -3 & 5 & 1 & 8 \end{bmatrix}$

Multiplying Matrices

To multiply an $m \times p$ and $p \times n$ matrix, note that the number of columns in the first matrix must be equal to the number of rows in the second matrix. The result will be an $m \times n$ matrix.

Let's begin by demonstrating what it means to multiply a row of one matrix by a column of another

For the matrices $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ $\quad B = \begin{bmatrix} 5 & -6 \\ 7 & 0 \end{bmatrix}$ $\quad R_1$ will denote row 1 of A

and C_1 will denote column 1 of B.

To multiply R_1 by C_1 means this: $R_1C_1 = \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 7 \end{bmatrix} = 1(5) + 2(7) = 19$

Ex 5: Determine each of these for A and B above.

a) R_1C_2 b) R_2C_1 c) R_2C_2

The result is $AB = \begin{bmatrix} R_1C_1 & R_1C_2 \\ R_2C_1 & R_2C_2 \end{bmatrix}$.

Ex 6: a) Write the matrix AB for Ex 5. b) Compute BA for the same matrices.

Matrix Multiplication: Suppose A is an $m \times p$ matrix and B is a $p \times n$ matrix. Let R_i denote the i^{th} row of A and C_j denote the j^{th} column of B. The **product of A and B**, denoted AB, is the matrix defined by

$$AB = \begin{bmatrix} R_1C_1 & R_1C_2 & \cdots & R_1C_n \\ R_2C_1 & R_2C_2 & \cdots & R_2C_n \\ \vdots & \vdots & & \vdots \\ R_mC_1 & R_mC_2 & \cdots & R_mC_n \end{bmatrix}$$

Ex 7: Perform multiplication on these.

a) $\begin{bmatrix} -3 & 4 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$ b) $\begin{bmatrix} 5 & 2 \\ 3 & -1 \end{bmatrix}^2$ c) $\begin{bmatrix} -1 & 3 & 6 \\ 4 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 6 & -2 \\ 3 & 5 & 1 \\ -3 & 4 & 0 \end{bmatrix}$

Properties of Matrix Multiplication Let A, B and C be matrices such that all of the matrix products below are defined and let r be a real number.

- **Associative Property of Matrix Multiplication:** $(AB)C = A(BC)$
- **Associative Property with Scalar Multiplication:** $r(AB) = (rA)B = A(rB)$
- **Identity Property:** For a natural number k, the $k \times k$ **identity matrix**, denoted I_k, is a square matrix containing 1's down the main diagonal and 0's elsewhere. For a $m \times n$ matrix A, $I_mA = AI_n = A$.
- **Distributive Property of Matrix Multiplication over Matrix Addition:**

$$A(B \pm C) = AB \pm AC \text{ and } (A \pm B)C = AC \pm BC$$