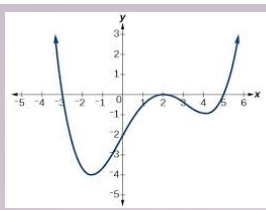
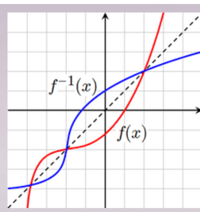
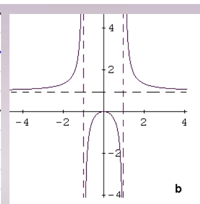
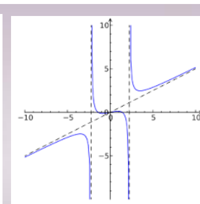
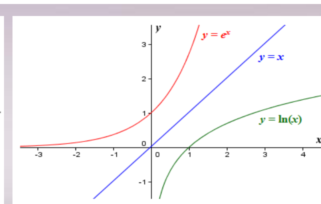










Math 1050 ~ College Algebra

21 Systems of Linear and Non-Linear Equations

$$\begin{aligned} -3x + 4y &= 5 \\ 2x - y &= -10 \end{aligned}$$

$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$

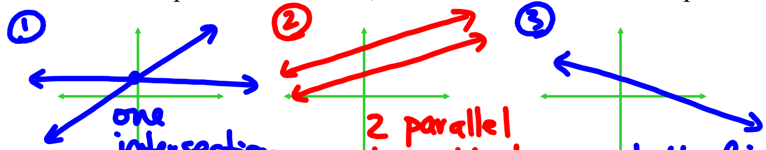
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

Learning Objectives

- Solve systems of two linear equations in two variables using substitution.
- Solve systems of two linear equations in two variables using elimination.
- Interpret solutions to 2×2 systems of linear equations.
- Solve systems of two non-linear equations in two variables using elimination.
- Solve systems of two non-linear equations in two variables using substitution.
- Solve and interpret solutions to 2×2 systems of non-linear equations.

A **system of equations** is simply more than one equation with two or more variables that we solve simultaneously.

If the two equations are linear, then one of three results is possible.



1. one intersection pt (one solution)
2. 2 parallel lines that never intersect (no solutions)
3. both lines are the same line (infinitely many solutions all the pts on the line)

Solving Strategies

1. Graph
2. Substitution
3. Elimination

(2. and 3.: most preferred)

4. Other methods

Ex 1: Solve using substitution.

a) $x - y = -4$
$x + 2y = 5$

a system of linear eqns in 2 variables

① choose either eqn and one variable to solve for
② solve for that variable in chosen eqn
③ plug in expression for chosen variable in other eqn
④ finish solving

① $x - y = -4$
② $x + 2y = 5$

choose ①, solve for x

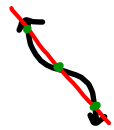
① $x = y - 4$
② $y - 4 + 2y = 5$
$3y = 9$
$\boxed{y = 3}$

$\Rightarrow$ ① $\boxed{x = 3 - 4 = -1}$

Solution: $\boxed{(-1, 3)}$

b) ① $3x + y = 2$
② $x^3 - 2 + y = 0$

a system of non-linear eqns in 2 variables



① $y = -3x + 2$
② $y = -x^3 + 2$

① solve for y.
$y = -3x + 2$

② $x^3 - \cancel{2} + (-3x + \cancel{2}) = 0$
$x^3 - 3x = 0$
$x(x^2 - 3) = 0$

$x = 0$ $\quad$ $x^2 - 3 = 0$
$x^2 = 3$
$x = \pm\sqrt{3}$

$x = 0$	$x = \sqrt{3}$	$x = -\sqrt{3}$
$y = -3(0) + 2$	$y = -3\sqrt{3} + 2$	$y = -3(-\sqrt{3}) + 2$
$y = 2$		$y = 3\sqrt{3} + 2$

solutions:
$\boxed{(0, 2), (\sqrt{3}, -3\sqrt{3} + 2), (-\sqrt{3}, 3\sqrt{3} + 2)}$

Ex 2: Solve by graphing, then by substitution.

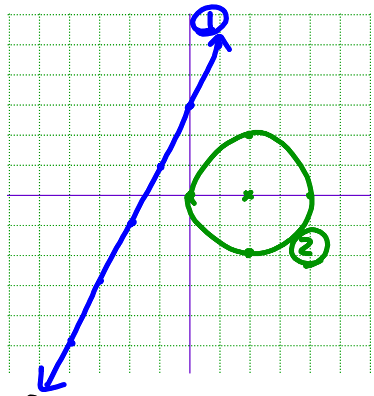
① $2x - y + 3 = 0$ $\quad y = 2x+3$

② $x^2 + y^2 - 4x = 0$

$(x^2 - 4x + \underline{4}) - \underline{4} + y^2 = 0$

$(x-2)^2 + y^2 = 4$

circle w/ center at $(2,0)$
$r = 2$



$\boxed{\text{No solution}}$

① $y = 2x+3$

② $x^2 + y^2 - 4x = 0$

② $x^2 + (2x+3)^2 - 4x = 0$

$x^2 + 4x^2 + 6x + 6x + 9 - 4x = 0$

$5x^2 + 8x + 9 = 0$

$x = \dfrac{-8 \pm \sqrt{64 - 4(5)(9)}}{2(5)}$ negative number

$\Rightarrow$ no valid x-value

$\Rightarrow \boxed{\text{No solution}}$

Ex 3: Solve by Elimination.

a) $3x - 2y = 7$
$\quad 8x + 4y = 0$

① eqns are listed in same order

② add straight down to eliminate a variable

OR might have to "scale" one or both eqns first to get a matching (but opposite sign) term

③ finish solving

① $(3x - 2y = 7)2$

② $8x + 4y = 0$

①a $6x - 4y = 14$

② $+ \; 8x + 4y = 0$

$14x = 14$

$x = 1$

① $3(1) - 2y = 7$

$3 - 2y = 7$

$-2y = 4$

$y = -2$

solution: $\boxed{(1,-2)}$

b) $x^2 + 3y = 6$ ①
$\quad y^2 - x^2 = 4$ ②

① $x^2 + 3y = 6$

② $+ \; -x^2 + y^2 = 4$

$3y + y^2 = 10$

$y^2 + 3y - 10 = 0$

$(y+5)(y-2) = 0$

$y + 5 = 0 \quad y - 2 = 0$

$y = -5$ or $y = 2$

① $x^2 - 15 = 6$ | $x^2 + 6 = 6$

$x^2 = 21$ | $x^2 = 0$

$x = \pm\sqrt{21}$ | $x = 0$

solutions: $\boxed{(\sqrt{21}, -5), (-\sqrt{21}, -5), (0, 2)}$

Ex 4: Solve algebraically by a method of your choice.

a) $5(5x - 3y = -2)$ ①
$3(3x + 5y = 9)$ ②

Elimination method

$25x - 15y = -10$
$+\ 9x + 15y = 27$
$34x = 17$
$x = \frac{1}{2}$

① $5\left(\frac{1}{2}\right) - 3y = -2$
$-3y = -2 - \frac{5}{2}$
$\frac{-1}{3} \cdot -3y = \frac{-9}{2} \cdot \frac{-1}{3}$
$y = \frac{3}{2}$

Solution: $\boxed{\left(\frac{1}{2}, \frac{3}{2}\right)}$

b) $3y = 4x - 5$ ①
$-8x + 6y = 1$ ②

Substitution method

② $-8x + 2(3y) = 1$
$-8x + 2(4x - 5) = 1$
$-\cancel{8x} + \cancel{8x} - 10 = 1$
$-10 = 1$
false statement
$\Rightarrow$ $\boxed{\text{No solution}}$
(these lines are parallel)

c) $9x - 3y = -15$ ①
$-3x + y = 5$ ②

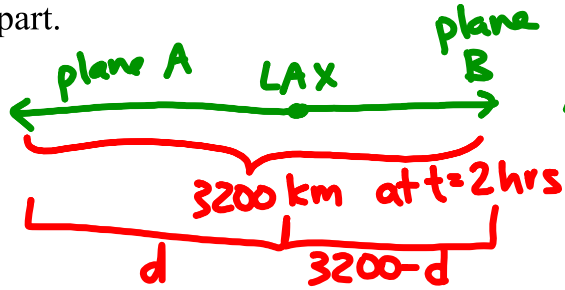
Substitution method

② $y = 3x + 5$
① $9x - 3(3x + 5) = -15$
$\cancel{9x} - \cancel{9x} - 15 = -15$
$-15 = -15$
true statement

$\boxed{\text{infinitely many solutions, i.e. these are the same line}}$

Application

Ex 5: Two planes start from LAX and fly in opposite directions. The second plane starts a half-hour after the first plane, but its speed is 80 kph faster. Find the airspeed of each plane if 2 hours after the first plane departs the planes are 3200 km apart.



	r	· t	= d
A	r	2	d
B	r+80	$1\frac{1}{2}=\frac{3}{2}$	3200-d

A: $2r=d$

B: $(r+80)\frac{3}{2}=3200-d$

B: $\frac{3}{2}r+80\left(\frac{3}{2}\right)=3200-2r$

$\frac{3}{2}r+120=3200-2r$

$\frac{2}{7}\cdot\frac{7}{2}r=3080\cdot\frac{2}{7}$

Speed of plane A $\boxed{r=880\text{ kph}}$

Speed of plane B

$r+80=\boxed{960\text{ kph}}$