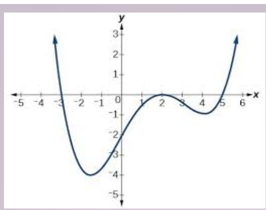
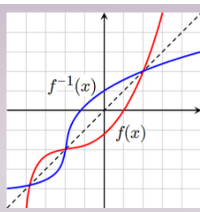
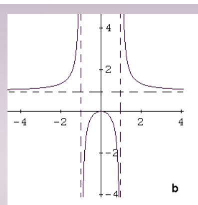
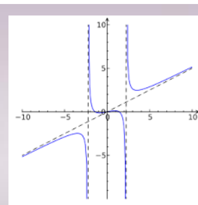
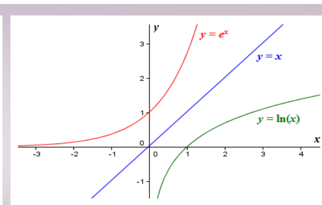










Math 1050 ~ College Algebra

15 Solving Rational Equations and Inequalities

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

Learning Objectives

- Solve rational equations.
- Solve rational inequalities graphically.
- Solve rational inequalities algebraically.

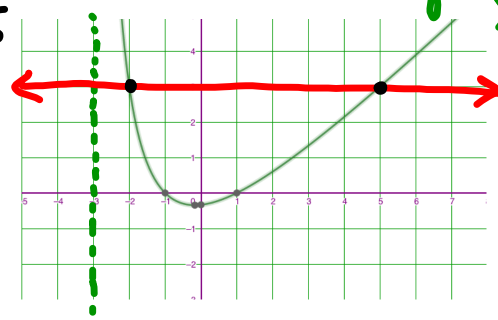
Solving Rational Equations and Inequalities

Ex 1: For each of these equations, determine the solution from the graph, then do the algebra to arrive at the same answer.

a) $\dfrac{x^2-1}{x+3}=3$

$x=-2,5$

$y=\dfrac{x^2-1}{x+3}$



$y=3$

$x^2-1=3(x+3)$

$x^2-1=3x+9$

$x^2-3x-10=0$

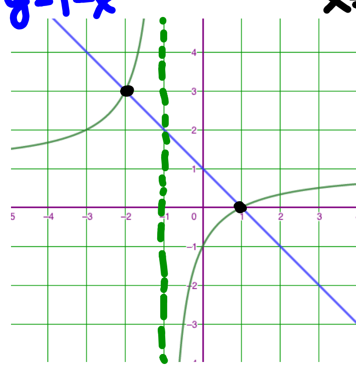
$(x-5)(x+2)=0$

$x-5=0$ or $x+2=0$

$\boxed{x=5,-2}$

b) $\dfrac{x-1}{x+1}=1-x$

$y=1-x$ $\qquad x=-2,1$



$y=\dfrac{x-1}{x+1}$

$x-1=(1-x)(x+1)$

$\cancel{x}-1=\cancel{x}+1-x^2-x$

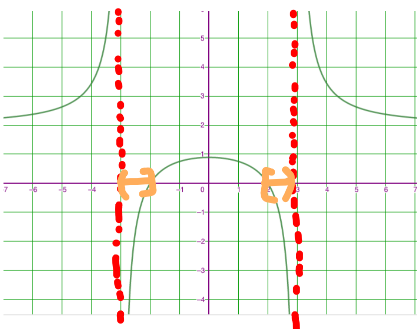
$x^2+x-2=0$

$(x-1)(x+2)=0$

$x-1=0$ or $x+2=0$

$\boxed{x=1,-2}$

Ex 2: Determine the solution graphically and algebraically.



$y = \dfrac{2x^2-8}{x^2-9}$

$$\frac{2x^2-8}{x^2-9} \le 0$$

where are the y-values negative or zero?

graphically: $(-3,-2] \cup [2,3)$

To solve algebraically

1. get everything on one side of inequality sign, w/ zero on other side
2. completely factor numerator & denominator
3. fill in sign line (x-values that make num. or den. $= 0$ go on sign line)
4. use sign line info to answer given question

algebraically:

$$\frac{2x^2-8}{x^2-9} \le 0$$

$$\frac{2(x-2)(x+2)}{(x-3)(x+3)} \le 0$$

$\boxed{(-3,-2] \cup [2,3)}$

sign line: $+$ -3 $-$ -2 $+$ 2 $-$ 3 $+$

test values:

① $x=-1000$ $\quad \dfrac{+(-)(-)}{-(-)}$

② $x=-2.5$ $\quad \dfrac{+(-)(-)}{-(+)}$

③ $x=0$ $\quad \dfrac{+(-)(+)}{-(+)}$

④ $x=2.5$ $\quad \dfrac{+(+)(+)}{-(+)}$

⑤ $x=1000$ $\quad \dfrac{+(+)(+)}{+(+)}$

Ex 3: Solve algebraically.

a) $\dfrac{3x}{x-1} \ge \dfrac{x}{x+4} + 3$

$$\frac{3x}{x-1} - \frac{x}{x+4} - 3 \ge 0$$

$$\frac{3x}{x-1}\left(\frac{x+4}{x+4}\right) - \frac{x}{x+4}\left(\frac{x-1}{x-1}\right) - 3\left(\frac{(x-1)(x+4)}{(x-1)(x+4)}\right) \ge 0$$

$$\frac{3x^2+12x-x^2+x-3(x^2+3x-4)}{(x-1)(x+4)} \ge 0$$

$$\frac{2x^2+13x-3x^2-9x+12}{(x-1)(x+4)} \ge 0$$

$$\frac{-x^2+4x+12}{(x-1)(x+4)} \ge 0$$

$$\frac{(-x+6)(x+2)}{(x-1)(x+4)} \ge 0$$

solution: $\boxed{(-4,2] \cup (1,6]}$

sign line: $-$ | -4 | $+$ | -2 | $-$ | 1 | $+$ | 6 | $-$

test values:

① $x=-1000$ $\quad \dfrac{+(-)}{-(-)}$

② $x=-3$ $\quad \dfrac{+(-)}{-(+)}$

③ $x=0$ $\quad \dfrac{+(+)}{-(+)}$

④ $x=2$ $\quad \dfrac{+(+)}{+(+)}$

⑤ $x=1000$ $\quad \dfrac{-(+)}{+(+)}$

b) $\dfrac{(x-2)(x+1)^2}{x(x+1)} \ge 0$

solution: $(-1,0) \cup [2,\infty)$

sign line: $-$ | -1 | $+$ | 0 | $-$ | 2 | $+$

test values:

① $x=-1000$ $\quad \dfrac{-(+)}{-(-)}$

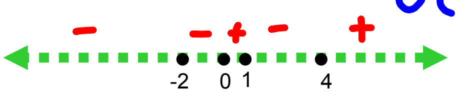
② $x=-\frac{1}{2}$ $\quad \dfrac{-(+)}{-(+)}$

③ $x=1$ $\quad \dfrac{-(+)}{+(+)}$

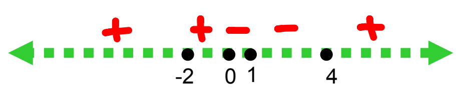
④ $x=1000$ $\quad \dfrac{+(+)}{+(+)}$

Ex 4: For each of these inequalities, fill in a sign line.

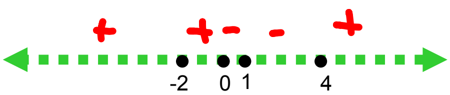
a) $\dfrac{3x(x-4)}{(x-1)(x+2)^2} \le 0$ $(-\infty,-2)\cup(-2,0]$ $\cup(1,4]$



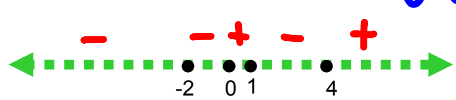
b) $\dfrac{3x(x-4)^3}{(x-1)^2(x+2)^2} \le 0$ $[0,1)\cup(1,4]$



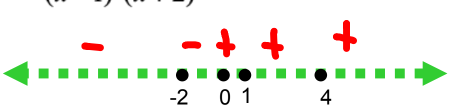
c) $\dfrac{3x(x-4)}{(x-1)^2(x+2)^2} \le 0$ $[0,1)\cup(1,4]$



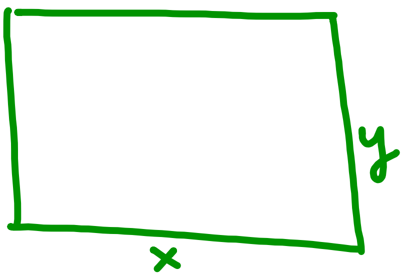
d) $\dfrac{3x(x-4)}{(x-1)^3(x+2)^2} \le 0$ $(-\infty,-2)\cup(-2,0]$ $\cup(1,4]$



e) $\dfrac{3x(x-4)^2}{(x-1)^2(x+2)^2} \le 0$ $(-\infty,-2)\cup(-2,0]$



Ex 5: A rectangular parking lot with a perimeter of 360 m is to have an area of at least 8000 m^2. Within what bounds must the length of the rectangle be?



$2x+2y=360$

$x+y=180$

$y=180-x$

$A=xy$

$xy\geq 8000$

solve for x.

$xy \geq 8000$

$x(180-x)\geq 8000$

$180x-x^2\geq 8000$

$0\geq x^2-180x+8000 \iff x^2-180x+8000\leq 0$

$(x-80)(x-100)\leq 0$

Solution:

$\boxed{x\in[80,100] \text{ meters}}$