

Math 1050 ~ College Algebra

14 Graphs with Holes and Variations on Asymptotes

Learning Objectives

- Identify holes in the graph of a rational function.
- Graph rational functions without vertical asymptotes.
- Find slant (oblique) asymptotes.
- Graph rational functions having slant asymptotes.

$$\begin{aligned} -3x + 4y &= 5 \\ 2x - y &= -10 \end{aligned}$$

$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^m k = \frac{m(m+1)}{2}$$

$$\sum_{k=0}^n z^k = \frac{1-z^{n+1}}{1-z}$$

Graph 1: A cubic function $f(x)$ and its inverse $f^{-1}(x)$ are shown, illustrating the property that the graph of a function and its inverse are symmetric with respect to the line $y=x$.

Graph 2: A rational function with a vertical asymptote at $x=2$ and a horizontal asymptote at $y=1$.

Graph 3: A rational function with a vertical asymptote at $x=1$ and a horizontal asymptote at $y=2$.

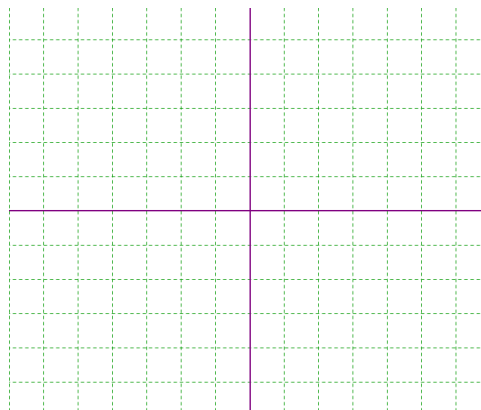
Graph 4: A rational function with a slant asymptote $y=x$ and a horizontal asymptote $y=2$.

Graph 5: A rational function with a horizontal asymptote at $y=2$ and a vertical asymptote at $x=1$.

Since there can be no points on the vertical asymptotes, what happens in an example like this?

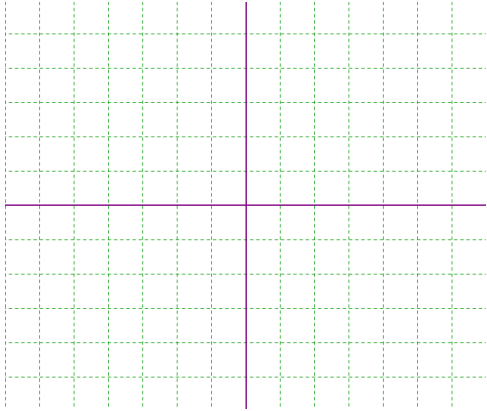
Ex 1: Analyze and graph.

$$H(x) = \frac{x-2}{x^2-4}$$



Graphing Rational Functions with No Vertical Asymptotes

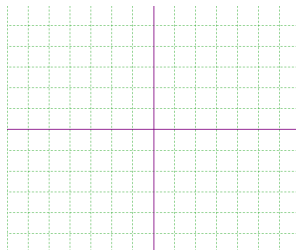
Ex 2: Analyze and graph. $H(x) = \frac{2x+3}{x^2+2}$



Identifying Slant (Oblique) Asymptotes

Ex 3: Analyze and graph.

$$H(x) = \frac{x^2 - x + 6}{x + 3}$$



Ex 4: Analyze and graph.

$$f(x) = \frac{x^3 - 1}{x - 1}$$

