Math 1050 ~ College Algebra

$-3x + 4y = 5$
$2x - y = -10$
$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$

$\sum_{i=1}^{m} k = \dfrac{m(m+1)}{2}$

$\sum_{k=0}^{n} z^k = \dfrac{1 - z^{n+1}}{1 - z}$

12 Introduction to Rational Functions

Learning Objectives

- Identify a rational function.
- Determine the domain of a rational function.
- Find the *x*- and *y*-intercepts for a rational function.
- Identify vertical and horizontal asymptotes.
- Graph irreducible rational functions with constant or first degree numerators and denominators of degree one.

A rational function is a ratio of two polynomial functions.
$f(x) = \dfrac{N(x)}{D(x)}$ where N(*x*) and D(*x*) are polynomials.

Ex 1: Determine which of these functions are rational functions.

a) $f(x) = \dfrac{x^2 + 1}{x + 4}$
b) $f(x) = \dfrac{3x + 2}{\sqrt{x} - 3}$
c) $f(x) = \dfrac{x^2 - 2x - 3}{\pi}$
d) $f(x) = \dfrac{x^{2.5} + 5}{x^2 - 25}$

Vertical Asymptotes of Simplified Rational Functions
- determined by finding disallowed denominator values
- line $x = a$ where D(a) = 0
- graph will never cross or touch

Ex 2: Find the domain and the vertical asymptotes for these functions.

a) $f(x) = \dfrac{2x^2}{x^2 - 1}$
b) $f(x) = \dfrac{3x}{x^2 + 1}$
c) $f(x) = \dfrac{x + 4}{4x - 2x^2}$

Horizontal Asymptotes
- end behavior of the graph
- line $y = b$ where $\lim\limits_{x \to \pm\infty} f(x) = b$
- graph may cross it
- depends on degree of $N(x)$ and $D(x)$
  - ▴ degree $(N(x)) <$ degree $(D(x))$, $y = 0$
  - ▴ degree $(N(x)) =$ degree $(D(x))$, $y =$ ratio of the leading coefficients.

Ex 3: Find the horizontal asymptotes of these functions.

a) $f(x) = \dfrac{2x^2}{x^2 - 1}$

b) $f(x) = \dfrac{3x}{x^2 + 1}$

c) $f(x) = \dfrac{x + 4}{4x - 2x^2}$

Ex 4: For each of these functions, determine the x and y-intercepts, vertical and horizontal asymptotes and draw a quick sketch.

a) $f(x) = \dfrac{2x^2}{x^2 - 1}$

b) $f(x) = \dfrac{3x}{x^2 + 1}$

c) $f(x) = \dfrac{x + 4}{4x - 2x^2}$