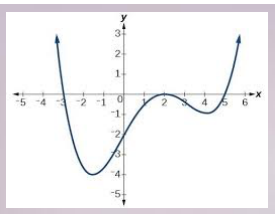
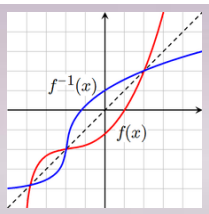
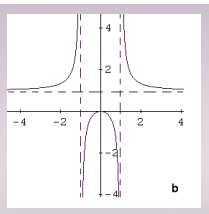
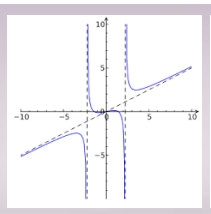
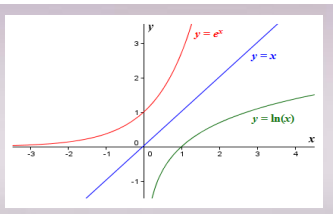





Math 1050 ~ College Algebra

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

11 Polynomial Inequalities

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

Learning Objectives

- Solve polynomial inequalities graphically.
- Solve polynomial inequalities analytically.

Graphical Interpretations of Equations and Inequalities

$\in$ = element of
$\cup$ = union (or)

Ex 1: Given this graph of f(x) and g(x), determine the values of x for which each of these is true.

a) $f(x) = g(x)$

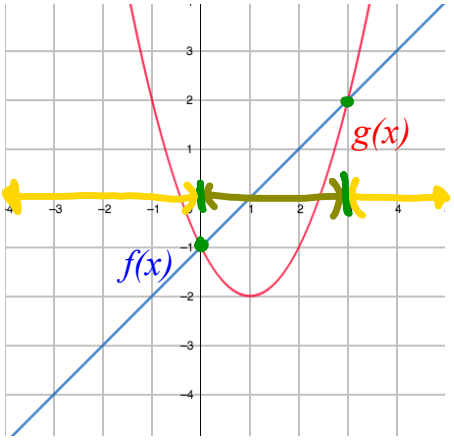
at $x = 0$ and $x = 3$

b) $f(x) < g(x)$ (line is below parabola)

$x \in (-\infty, 0) \cup (3, \infty)$

c) $f(x) > g(x)$ (line is above parabola)

$x \in (0, 3)$



Analytical Solution of Polynomial Inequalities

Ex 2: Given $f(x) = x^2 - 4$ and $g(x) = x + 2$, determine the values of x for which each of these is true by doing the math.

a) $f(x) = g(x)$

$x^2 - 4 = x + 2$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x - 3 = 0 \quad x + 2 = 0$

$x = 3, -2$

b) $f(x) < g(x)$ region ②

$x \in (-2, 3)$

sign line:

test cases:

① $x = -10$ $f(-10) = 100 - 4 = 96$ $g(-10) = -10 + 2 = -8$

② $x = 0$ $f(0) = 0 - 4 = -4$ $g(0) = 0 + 2 = 2$

③ $x = 5$ $f(5) = 5^2 - 4 = 21$ $g(5) = 5 + 2 = 7$

c) $f(x) > g(x)$ regions ① + ③

$x \in (-\infty, -2) \cup (3, \infty)$

As the functions get more complicated, it is convenient to use a **sign line** to sort it out.

Directions for Using a Sign Line

a) Write the inequality as a function, f, with zero on the right side.

b) Determine the zeros of f and place them on a number line.

c) Choose a test value in each of the intervals on the number line.

d) Determine the sign of f for each test value, writing that sign above that interval.

e) Your solution is the interval(s) that correspond to the inequality.

Ex 3: Follow the steps above to solve these inequalities.

a) $x^2 + 2x > 3$

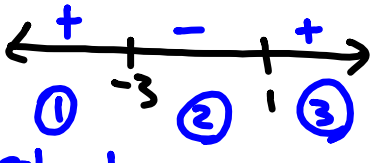
$x^2+2x-3>0$

$(x+3)(x-1)>0$ ① + ③

$x=-3, 1$

$x\in(-\infty,-3)\cup(1,\infty)$

sign line



test pts:

① $x=-1000$ $(-1000+3)(-1000-1)$ $-(-)$

② $x=0$ $(0+3)(0-1)$ $+(-)$

③ $x=1000$ $(1000+3)(1000-1)$ $+(+)$

b) $-3x^2 - 2x \geq -x^2 + x - 2$

$-2x^2-3x+2\geq 0$

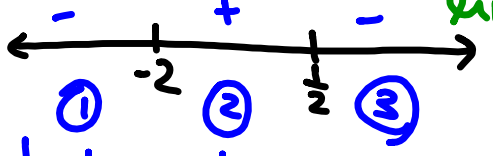
$(2x-1)(-x-2)\geq 0$ ②

$2x-1=0$ $\quad -x-2=0$

$x=\frac{1}{2}$ $\quad x=-2$

$x\in\left[-2,\frac{1}{2}\right]$

sign line



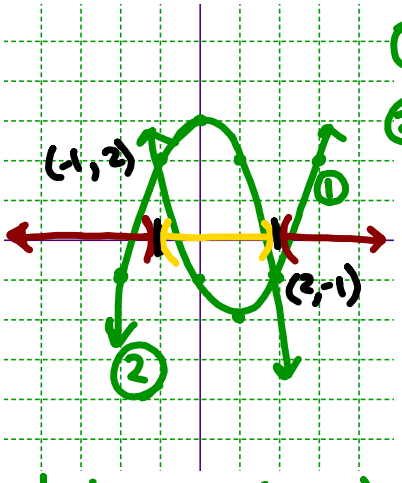
test x-values:

① $x=-1000$ $-(+)$ | ② $x=0$ $-(-)$ | ③ $x=1000$ $+(-)$

Ex 4: Solve this inequality by each method.

a) Graphically $(x\text{-}1)^2 - 2 > -x^2 + 3$

b) Analytically $(x\text{-}1)^2 - 2 > -x^2 + 3$



① $y = (x-1)^2 - 2$

② $y = -x^2 + 3$

y-values for ① > y-values for ②

claim: $x \in (-\infty, -1) \cup (2, \infty)$

we get the same answer both ways!

Note of warning: graphing doesn't always work so nicely, especially if your x-values are not integers.

$(x-1)(x-1)$

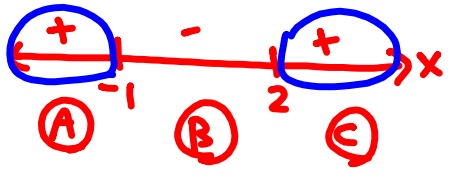
$x^2 - x - x + 1 - 2 > -x^2 + 3$

$2x^2 - 2x - 4 > 0$

$2(x^2 - x - 2) > 0$

$2(x-2)(x+1) > 0$

critical values: $x = 2, -1$



test pts:

A) $x = -1000000$

$-(-)$

B) $x = 0$

$-(+)$

C) $x = 1000$

$+(+)$