9.5 The Binomial Theorem

* Use the Binomial Theorem to calculate binomial coefficients.
* Use Pascal's Triangle to calculate binomial coefficients.
* Find the nth term in a binomial expansion.

What does the word binomial mean?

$(a+b)^0$

$(a+b)^1$

$(a+b)^2$

$(a+b)^3$

What does 7! mean?

Example 1 : Determine the value of each of these.

a) $4!$

b) $10!$

c) $12!/10!$

d) $n!$

e) $(n+2)!$

f) $0!$

Example 2: A pizza shop offers 4 different toppings, Onions, Mushrooms, Pepperoni and Ham. How many 'different' pizzas can you order having none, one, two, three or all four toppings?

What does ${}_nC_r$ mean?

${}_nC_r = \frac{n!}{(n-r)!r!} = \binom{n}{r}$

Determine the value of each of these.

${}_4C_0$ ${}_4C_1$ ${}_4C_2$ ${}_4C_3$ ${}_4C_4$

Example 3: Determine the value of each of these and make up a question it might answer.

${}_6C_2$

${}_{12}C_{10}$

${}_7C_4$

${}_{15}C_0$

Binomial Theorem and Pascal's Triangle

$(x+y)^n=$

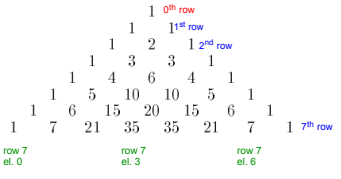
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$\binom{0}{0}$
$\binom{1}{0}$ $\binom{1}{1}$
$\binom{2}{0}$ $\binom{2}{1}$ $\binom{2}{2}$
$\binom{3}{0}$ $\binom{3}{1}$ $\binom{3}{2}$ $\binom{3}{3}$
$\binom{4}{0}$ $\binom{4}{1}$ $\binom{4}{2}$ $\binom{4}{3}$ $\binom{4}{4}$
$\binom{5}{0}$ $\binom{5}{1}$ $\binom{5}{2}$ $\binom{5}{3}$ $\binom{5}{4}$ $\binom{5}{5}$

So, $(a+b)^5 =$

Example 4: Expand this binomial. $(2x - y)^4 =$

Pascal's Triangle



Example 5: How do we find the $x^{6\text{th}}$ term in the expansion of $(2x\text{-}y)^{10}$ without writing the entire expansion?

Example 6: An interesting application of Pascal's Triangle is in probability. In a family of six children, what is the probability that two are boys and the rest are girls?