9.2 Arithmetic Sequences and Series

In section 9.2 you will learn to:

- Recognize, write and find the nth terms of arithmetic sequences.
- Find the nth partial sums of arithmetic sequences.
- Use arithmetic sequences to model and solve real-life problems.

A sequence $a_1, a_2, a_3, \ldots, a_n$ is said to be *arithmetic* is the difference d between consecutive terms remains constant.

Which of these are arithmetic sequences?

1, 3, 5, 7, ...

2, 4, 8, 16, ...

1, 4, 9, 16, ...

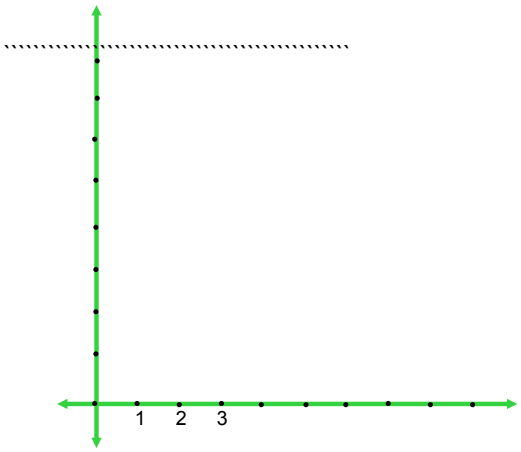
19, 11, 3, -5, -13, ...

Example 1:

Find the next three terms of the arithmetic sequence 1, 5, 9, 13, ...

Then, find a formula for the n^{th} term and use that to calculate a_{100}.

How would you describe the graph of an arithmetic sequence?



Example 2:

Suppose the 4^{th} term of an arithmetic sequence is 20 and the 13^{th} term is 65. What are the first six terms of the sequence?

Example 3:

A local theatre has a large auditorium with 22 rows of seats. There are 18 seats on Row 1 and each row after Row 1 has two more seats than the previous row. How many seats are in Row 22?

A *finite arithmetic series* is the sum S_n of the first n terms of a finite arithmetic sequence.

$$Sn = a_1 + (a_1+d) + (a_1 + 2d) + (a_1 + 3d) + ... + (a_1 + (n-1)d)$$

S_n can be found by computing $S_n = \frac{n}{2}(a_1 + a_n)$.

An alternate formula for S_n is $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

Example 4:

Use the summation notation to write these series and find the sums using either of the formulas:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

or

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

a) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17:

b) $\sum_{j=1}^{30} (9+3j)$

Example 5:

In the theatre we described previously, there were 22 rows of seating. There were 18 seats on Row 1 and each subsequent row had two more seats than the previous row.

What is the seating capacity of the auditorium?

Example 6:

In the last lesson, you decided to save for your trip to Europe. You opened a savings account with $1.00 and on each subsequent day, you deposited a dollar more than on the previous day.

How much have you contributed by the end of one year?