CHAPTER 9: SEQUENCES AND SERIES

9.1 Sequences and Series

In section 9.1 you will learn to:

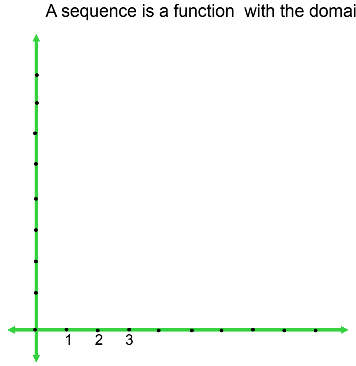
- Use sequence notation to write the terms of a sequence.
- Use factorial notation.
- Use summation notation to write sums.
- Find the sums of infinite series.
- Use sequences and series to model and solve real-life problems.

What is a sequence?

Finite: 1, 2, 4, 8

Infinite: 1, 3, 5, 7, ..., 2n-1, ...

A sequence is a function with the domain a subset of the natural numbers.



Example 1:

a) Write the first four terms of this sequence: $a_n = n^2 + 1$

b) Write the first four terms of this sequence: $b_n = (-1)^{n+1}(10n + 3)$

Example 2: Find a formula for the n^{th} term in each of these sequences, then use the formula to find the 10^{th} term.

a) 2, 4, 6, 8, 10, ...

b) 3, -6, 12, -24, ...

Some sequences are defined ***recursively***. One or more initial terms are given and subsequent terms are defined using the previous terms.

Example 3:

$a_1 = 2$ $a_n = 3a_{n-1} + 1$ for each $n > 1$

What are the first four terms?

Example 4:

The ***Fibonacci Sequence***

$a_1 = 1$

$a_2 = 1$

$a_k = a_{k-1} + a_{k-2}$

List five terms:

Factorials are often used in sequence definitions.

We define *n factorial* (written *n!*) to be:

$n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n$

$4! =$

$0!$ is defined to be $0! = 1$

Example 5:

Evaluate these expressions:

a) $\frac{8!}{2! \cdot 6!}$

b) $\frac{(n+1)!}{(n-1)!}$

It is often convenient to recognize the factorials of the first five or six natural numbers.

Example 6:

Write the first four terms of these sequences:

a) $a_n = \frac{1}{n!}$

b) $b_n = \frac{n}{(n+2)!}$

A ***series*** is the sum of the terms in a sequence. The sum of the first *n* terms of a sequence is the n^{th} *partial sum* S_n.

The 5^{th} partial sum of the sequence of odd numbers is S_5 =

For an arbitrary sequence $a_1, a_2, a_3, \dots, a_{100}$, the corresponding series is

$a_1 + a_2 + a_3 + \dots + a_{100}$.

We abbreviate this sum using the Greek letter Σ (sigma):

$$\sum_{i=1}^{100} a_i = a_1 + a_2 + a_3 + \dots + a_{100}.$$

The subscript i=1 and superscript 100 written above and below sign indicate which terms begin and end the series. The index i is not unique, but is sometimes replaced using j, k, etc.

Express $3^2 + 4^2 + 5^2 + 6^2$ using the sigma.

Example 7:

Find the sum of these series by adding the terms:

a) $\sum_{j=1}^{5} (1+3j)$

b) $\sum_{k=0}^{2} \frac{(-1)^k}{2^k}$

Use summation notation to abbreviate this series:

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \quad + \dots + \frac{1}{99\cdot 100}$$

Example 8:

You're a clever student. You've decided to save your money for a trip to Europe, but it will be expensive. You've decided to open a savings account today with $1. You plan to add more each day, 7 days a week, by depositing one more dollar each day than you did the previous day. Use summation notation to express the total amount you will have contributed at the end of one year: