CHAPTER 8: MATRICES AND DETERMINANTS

In section 8.1 you will learn to:

- Write a matrix and identify the order.
- Perform elementary row operations on matrices
- Use matrices and Gaussian elimination to solve systems of linear equations.
- Use matrices and Gauss-Jordan elimination to solve systems of linear equations.

Definition of a matrix:

$[a_{ij}] = A =$ [ ]

entry →

order →

row matrix →

column matrix →

We will use matrices to solve linear systems of equations.

system

$$\begin{aligned} 3x - 2y + z &= 5 \\ x + y + 2z &= 1 \\ -x - z &= 0 \end{aligned}$$

augmented matrix

$$\left[\begin{array}{ccc|c} & & & \\ & & & \\ & & & \end{array}\right]$$

A = X = C =

coefficient matrix variable matrix constant matrix

AX = C

Example 1 -- What is the size (order) of these matrices?
Are any of them square?

a) $\begin{bmatrix} -2 & 5 & 1 \\ 7 & 6 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 2 & 3 & 2 & 3 & 1 & 5 \\ 4 & 4 & 7 & 4 & 4 & -1 \\ 9 & 8 & 7 & 6 & 5 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$

Row-echelon form →
- All zero rows at the bottom
- Has a leading 1 in every nonzero row
- All entries below the leading 1 are zero.

$$\left[\begin{array}{ccc:c} 1 & 0 & 2 & 3 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \left[\begin{array}{ccc:c} 1 & 0 & 2 & 3 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \left[\begin{array}{ccc:c} 1 & 0 & 2 & 3 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Reduced row-echelon form →
- row-echelon form

and
- all entries above leading 1 are zero

$$\left[\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

Example 2 - Indicate if these matrices are in
- ⓐ - row-echelon form
- ⓑ - reduced row-echelon form
- ⓒ - neither

a) $\left[\begin{array}{ccc} 2 & 1 & 0 \\ 0 & 1 & 5 \\ 1 & 0 & 0 \end{array}\right]$ b) $\left[\begin{array}{cccc} 1 & 5 & 6 & 3 \\ 0 & 1 & 2 & -1 \end{array}\right]$ c) $\left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 1 & 2 & 7 & 9 \\ 0 & 1 & 3 & 5 \end{array}\right]$ d) $\left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right]$

Example 3: a) Write the system of equations represented by this augmented matrix.
b) Write this matrix in row-echelon form.
c) Back-substitute to solve.

$$\left[\begin{array}{ccc:c} 1 & 0 & -3 & -2 \\ 3 & 1 & -2 & 5 \\ 2 & 2 & 1 & 4 \end{array}\right]$$